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ON REMOVABLE SETS OF SOLUTIONS OF DEGENERATE ELLIPTIC EQUATIONS WITH MINOR TERMS

O.S.ALIYEV
Institute of Mathematics and Mechanics of NAS of Azerbaijan
soltanaliyev@yahoo.com

In this paper we consider the nondivergent degenerate elliptic equations of the second order with minor terms. The sufficient condition of removability of compact with respect to these equations in the weight space of Hölder functions was found.

Key words: removable, solution, degenerate, equation

The aim of the given paper is finding sufficient condition of removability of compact with respect to the equation in the space $C_\omega^\lambda(D)$. This problem has been investigated by many degenerate elliptic researchers. For the Laplace equation the corresponding result was found by L. Carleson [1]. Concerning the second order elliptic equations of divergent structure, we show in this direction the papers [2], [3]. For a class of non-divergent elliptic equations of the second order with discontinuous coefficients of removability condition for a compact in the space $C^\lambda(D)$ was found in [4]. Mention also papers [5-7], in which the conditions of removability for a compact in the space of continuous functions have been obtained. The removable sets of solutions of the second order elliptic and parabolic equations in nondivergent form was considered in [10]-[12]. In [13], T. Kilpelainen and X. Zhong have studied the divergent quasilinear equation without minor members proved the removability of compact. Removable sets for pointwise solutions of elliptic partial differential equations was found by J. Diederich [14]. Removable singularities of solutions of linear partial differential equations was considered in R.Harvey, J.Polking [15]. On removable sets at the boundary for subharmonic functions has been investigated by B. Dahlberg [16].

Let D be a bounded domain situated in n dimensional Euclidean space E_n of points $x=(x_1,...,x_n),\quad n\geq 3,\ \ \partial D$ be its boundary. Consider in D the following elliptic equation

$$L_u=\sum_{i,j=1}^{n}a_{ij}(x)u_{ij}+\sum_{i=1}^{n}b_i(x)u_i+c(x)u=0 \qquad (1)$$

in supposition that $\|a_{ij}(x)\|$ is a real symmetric matrix, moreover $\omega(x)$ be a positive measurable function satisfying the doubling condition: for concentric balls B_R^x or R and $2R$ radius, there exists such a constant γ

$$\omega(B_R^x)\geq\gamma\omega(B_{2R}^x),$$

where for the measurable sets $E\omega(E)$ means $\int_E\omega(y)dy$

$$\gamma|\xi|^2\omega(x)\leq\sum_{ij=1}^{n}a_{ij}(x)\xi_i\xi_j\leq\gamma^{-1}\omega(x)|\xi|^2;\qquad \xi\in E_n,\quad x\in D, \qquad (2)$$

$$a_{ij}(x)\in C_\omega^1(\overline{D});\qquad i,j,1,\ldots,n, \qquad (3)$$

$$|b_i(x)\leq b_0;|\quad -b_0\leq c(x)\leq 0;\quad i=1,\ldots,n;\quad x\in D. \qquad (4)$$

Here $u_i=\dfrac{\partial u}{\partial x_i}$, $u_{ij}=\dfrac{\partial^2 u}{\partial x_i\partial x_j}$; $i,j=1,\ldots,n$; $\gamma\in(0,1]$ and $b_0\geq 0$ are constants. Besides we'll suppose that the lower coefficients of the operator L are measurable functions in D. Let $\lambda\in(0,1)$ be a number. Denote by $C^{0,\lambda}(D)$ a Banach space of the functions $u(x)$ defined in D with the finite norm.

$$\|u\|_{C_\omega^\lambda(D)}=\sup_{x\in D}\omega(x)\left|u(x)+\sup_{\substack{x,y\in D\\ x\neq y}}\right|\frac{|u(x)-u(y)|\,\omega}{|x-y|^\lambda}.$$

The compact $E\subset\overline{D}$ is called removable with respect to the equation (1) in the space $C_\omega^\lambda(D)$ if from

$$L_u=0,\qquad x\in D\setminus E;\qquad u\Big|_{\partial D\setminus E}=0;\qquad u(x)\in C_\omega^\lambda(D) \qquad (5)$$

it follows that $u(x)\equiv 0$ in D.

Denote by $B_R(z)$ and $S_R(z)$ the ball $\{x:|x-z|<R\}$ and the sphere $\{x:|x-z|=R\}$ of radius R with the center at the point $z\in E_n$ respectively. We'll need the following generalization of mean value theorem belonging to E.M. Landis and M.L. Gerver [8] in weight case.

Lemma. Let the domain G be situated between the spheres $S_R(0)$ and $S_{2R}(0)$, moreover the intersection $\partial G\cap\{x:R<|x|<2R\}$ be a smooth surface. Further, let in G the uniformly positive determined matrix $\|a_{ij}(x)\|$; $i,j=1,\ldots,n$

and the function $u(x) \in C^2(G) \cap C^1_\omega(\overline{G})$ be given. Then there exists the piece-wise smooth surface $\sum$ dividing in G the spheres $S_R(0)$ and $S_{2R}(0)$ such that

$$\int_\Sigma \omega \left|\frac{\partial u}{\partial v}\right| ds \le K \underset{G}{osc}\, u \cdot \frac{\omega(G)}{R^2}$$

Here $K > 0$ is a constant, depending only on matrix $\|a_{ij}(x)\|$ and n, and $\frac{\partial u}{\partial v}$ is a derivative by conormals determined by the equality

$$\frac{\partial u(x)}{\partial v} = \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial u(x)}{\partial x_i} \cos(\overline{n}, x_j)^{\frac{1}{2}},$$

where $\cos(\overline{n}, x_j)$; $j = 1,...,n$ -are directing cosine of unit external normal vector to $\sum$.

Denote by $W^1_{2,\omega}(D)$ the Banach space of the functions $u(x)$ defined in D with the finite norm

$$\|u\|_{W^1_{2,\omega}(D)} = \left(\int_D \omega \left(u^2 + \sum_{i=1}^{n} u_i^2\right) dx\right)^{\frac{1}{2}}$$

and let $\overset{0}{W}{}^1_{2,\omega}(D)$ be a completion $C_0^\infty(D)$ by the norm of the space $W^1_{2,\omega}(D)$.

By $m_H^s(A)$ we'll denote the Hausdorff measure of the set A of order $s > 0$. Further everywhere the notation $C(...)$ means, that the positive constant C depends only on content of brackets.

Theorem 1. Let D be a bounded domain in E_n, $E \subset \overline{D}$ be a compact. If with respect to the coefficients of the operator L the conditions (1)-(3) are fulfilled, then for removability of the compact E with respect to the equation (5) in the space $C^\lambda_\omega(D)$ it sufficies that

$$m_H^{n-2+\lambda}(E) = 0 \tag{6}$$

Proof. At first we show that without loss of generality we can suppose the condition $\partial D \in C^1$ to be fulfilled. Suppose, that the condition (6) provides the removability of the compact E for the domains, whose boundary is the surface of the class C^1, but $\partial D \in C^1$ and by fulfilling (6) the compact E is not removable. Then the problem (5) has non-trivial solution $u(x)$, moreover $u\big|_E = f(x)$ and $f(x) \ne 0$. We always can suppose the lowest coefficients of the operator L to be infinitely differentiable in D. Moreover, without loss of generality, we'll suppose that the coefficients of the operator L are extended to a ball $B \supset \overline{D}$ with saving the conditions (2)-(4). Let $f^+(x) = \max\{f(x), 0\}$,

$f^-(x) = \min\{f(x), 0\}$ and $u^{\pm}(x)$ be generalized by Wiener (see [8]) solutions of the boundary value problems

$$L_u^{\pm} = 0,\ x \in D \setminus E;\ u^{\pm}\Big|_{\partial D \setminus E} = 0;\ \ u^{\pm}\Big|_E = f^{\pm}.$$

Evidently, by $u(x) = u^+(x) + u^-(x)$. Further, let D'-be such a domain, that $\partial D' \in C^1$, $\overline{D \subset D'}$, $\overline{D}' \subset B$, and $v^{\pm}(x)$ be solutions of the problems

$$Lv^{\pm} = 0,\ x \in D' \setminus E;\ \ v^{\pm}\Big|_{\partial D'} = 0;\ \ v^{\pm}\Big|_E = f^{\pm};\ v^{\pm}(x) \in C_\omega^\lambda(D').$$

By the maximum principle for $x \in D$

$$0 \le u^+(x) \le v^+(x), \quad v^-(x) \le u^-(x) \le 0.$$

But according to our supposition $v^+(x) \equiv v^-(x) \equiv 0$. Hence, it follows, that $u(x) \equiv 0$. So, we'll suppose, that $\partial D \in C^1$. Now, let $u(x)$ be a solution of the problem (5), and the condition (6) be fulfilled. Give an arbitrary $\varepsilon > 0$. Then there exists a sufficiently small positive number δ and a system of the balls $\{B_{rk}(x^k)\}$, $k = 1, 2, \ldots$, such that $r_k < \delta$, $E \subset \bigcup_{k=1}^{\infty} B_{rk}(x^k)$ and

$$\sum_{k=1}^{\infty} r_k^{n-2+\lambda} < \varepsilon. \tag{7}$$

Consider a system of the spheres $\{B_{2rk}(x^k)\}$, and let $D_k = D \cap B_{2rk}(x^k), k = 1, 2, \ldots, .$. Without loss of generality we can suppose, that the cover $\{B_{2rk}(x^k)\}$ has a finite multiplicity $a_0(n)$. By the Landis-Gerver theorem for every k there exists a piece-wise smooth surface $\sum k$ dividing in D_k the spheres $S_{rk}(x^k)$ and $S_{2rk}(x^k)$, such that

$$\int_{\sum_k} \omega \frac{\partial u}{\partial \nu} ds \le k \underset{D_k}{osc}\, u \frac{\omega(D_k)}{r_k^2}. \tag{8}$$

Since $u(x) \in C_\omega^\lambda(D)$, there exists a constant $H_1 > 0$, depending only on the function $u(x)$, such that

$$osc\, u\omega \le H_1 (2r_k)^\lambda \tag{9}$$

Besides

$$\omega(D_k) \le mes_n B_{2rk}(x^k) = \Omega_n 2^n r_k^n; \quad k = 1, 2, \ldots, \tag{10}$$

where $\Omega_n = mes_n B_1(0)$. Using (9)-(10) in (8) we get

$$\int_{\sum_k} \omega \frac{\partial u}{\partial \nu} ds \le C_1 r_k^{n-2+\lambda}; \ \ k = 1, 2, \ldots, \tag{11}$$

where $C_1 = KH_1 2^{n+\lambda}$.

Let D_Σ be an open set situated in $D \setminus E$, whose boundary consists of unification of Σ and Γ where $\Sigma = \bigcup\limits_{k=1}^{\infty} \Sigma_k$, $\Gamma = \partial D \setminus \bigcup\limits_{k=1}^{\infty} D_k^{+}$, D_k^+ is a part of D_k remaining after the removing of points situated between Σ and $S_{2r_k}(x^k)$; $k = 1, 2, \ldots$. Denote by $D_\Sigma^{'}$ the arbitrary connected component D_Σ, and by M we denote the ellipitic operator of divergent structure

$$M = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x) \frac{\partial}{\partial x_j} \right).$$

According to Green formula for any functions $z(x)$ and $W(x)$ pertaining to the intersection $C^2(D_\Sigma^{'} \cap C^1(\overline{D_\Sigma^{'}}$ we have

$$\int_{D_\Sigma^{'}} \left(zM\beta - \beta Mz \right) dx = \int_{\partial D_\Sigma^{'}} \left(z\frac{\partial \beta}{\partial \nu} - \beta \frac{\partial z}{\partial \nu} \right) ds . \qquad (12)$$

As is known $\partial D \in C^1$ then $u(x) \in C^1\left(D_\Sigma^{'}\right) \cap C^1\left(\overline{D_\Sigma^{'}}\right)$ (see [9]). From (12) choosing the functions $z = 1$, $\beta = \omega u^2$ we have

$$\int_{D_\Sigma^{'}} M\left(\omega u^2\right) dx = 2\int_{\partial D_\Sigma^{'}} \omega u \frac{\partial u}{\partial \nu} ds + \int_{\partial D_\Sigma} \omega_{x_i} u^2 ds .$$

But $|u(x)| \le M < \infty$ for $x \in \overline{D}$. Let's put the condition:

$$\omega_{x_i} < c\omega \qquad (*)$$

By virtue of condition (*) and $\int\limits_{\partial D_\Sigma} \omega u^2 ds < C_3 M_\varepsilon$, subject to (11) and (7) we conclude

$$\int_{\partial D_\Sigma} M\left(\omega u^2\right) dx \le 2Ma_0 \sum_{k=1}^{\infty} \int_{\Sigma_k} \omega \frac{\partial u}{\partial \nu} ds + \int_{D_\Sigma^{'}} \omega u^2 dx \le$$

$$\le 2Ma_0 C_1 \sum_{k=1}^{\infty} r_k^{n-2+\alpha} + \varepsilon M c_2 < C_3 \varepsilon , \qquad (13)$$

w $C_3 = 2Ma_0C_1$ here .

On the other side

$$\mathrm{M}\left(\omega u^2\right) = 6u\omega \mathrm{M}(u) +$$

$$+2\sum_{i,j=1}^{n} \omega a_{ij} u_i u_j + (2u+1)\sum_{i,j=1}^{n} a_{ij} u_{xj} \omega_{xi} + \sum_{i,j=1}^{n} \frac{\partial a_{ij}}{\partial x_i} u\omega_{xj} + \sum_{i,j=1}^{n} a_{ij} u \omega_{x_i x_j}$$

and besides,

$$M_u = L_u + \sum_{i=1}^{n} d_i(x) u_i - c(x) u,$$

where

$$d_i(x)=\sum_{j=1}^{n}\frac{\partial a_{ij}(x)}{\partial x_j}-b_i\left(x\right),\ \ i=1,...n\,.$$

It is evident, that by vistue of conditions (2)-(3) $\left|d_i(x)\right|\le d_0<\infty;\quad i=1,...n.$. Thus from (13) we obtain

$$6\int_{D'_{\Xi}}u\omega\sum_{i=1}^{n}d_i\left(x\right)u_i dx-6\int_{D'_{\Sigma}}u^2c\left(x\right)dx+2\int_{D'_{\Sigma}}\sum_{i,j=1}^{n}\omega\left(x\right)a_{ij}u_iu_jdx+$$

$$+\left(2u+1\right)\int_{D'_{\Sigma}}\sum_{i,j=1}^{n}a_{ij}u_j\omega_{x_i}dx+\int_{D'_{\Sigma}}\sum_{i,j=1}^{n}\frac{\partial a_{ij}}{\partial x_j}u\omega_{x_i}dx+\left|\nabla u\right|^2dx+$$

$$+\int_{D'_{\Sigma}}\sum_{i,j=1}^{n}a_{ij}u\omega_{x_ix_j}dx<C_{3\varepsilon}$$

Hence for any $\alpha>0$ it follows that

$$2\gamma\int_{D'_{\Sigma}}\omega\left|\nabla u\right|^2dx<6d_0\int_{D'_{\Sigma}}\omega\left|u\right|\left|u_i\right|dx+6\int_{D'_{\Sigma}}u^2\omega\left(x\right)+\left(2u+1\right)\int_{D'_{\Sigma}}a_{ij}u_j\omega_{x_i}dx+$$

$$+d_0\int_{D'_{\Sigma}}u\omega^2_{x_i}dx+\int_{D'_{\Sigma}}a_{ij}u\omega_{x_ix_j}+C_{3\varepsilon}\le 6\frac{d_0}{\varepsilon}\int_{D'_{\Sigma}}\left|u\right|^2dx+6\frac{d_0\varepsilon}{2}\int_{D'_{\Sigma}}\omega^2\left|\nabla u\right|^2dx+$$

$$+\left(2n+1\right)\int_{D'_{\Sigma}}u_j\omega dx+d_0\int_{D'_{\Sigma}}u\omega\,dx+\gamma C_{4\varepsilon}\le 6\frac{d_0}{\varepsilon}Mmes_nD+$$

$$+\frac{\left(2M+1\right)\gamma}{\varepsilon}mes_nD+d_0M\omega\left(D\right)+\gamma C_4M\omega\left(D\right)+C_3\varepsilon$$

If we'll take into account that

$$\left|\omega_{x_ix_j}\right|<C_4\omega\left(x\right),$$

then from here we have that

$$\int_{D'_{\Sigma}}\omega^2\left|\nabla u\right|^2dx\le C_5$$

where $C_5=\left(6d_0+\left(2M+1\right)\right)Mmes_nD+\left(d_0M+\gamma C_4M\right)\omega\left(D\right)+\frac{C_3}{\gamma}$. Without loss of generality we assume that $\varepsilon\le 1$. Hence we have

$$\int_{D}\omega^2\left|\nabla u\right|^2dx\le C_6$$

Thus $u\left(x\right)\in W^1_{2,\omega}\left(D\right)$. From the boundary condition and $mes_{n-1}\left(\partial D\cap E\right)=0$ we get $u\left(x\right)\in \overset{\circ}{W}{}^1_{2,\omega}\left(D\right)$. Now, let σ be a number, which will be chosen later, $D^+_{\Sigma}=\left\{x:x\in D'_{\Sigma},\,u\left(x\right)>0\right\}$. Without loss of generality, we suppose, that the set D^+_{Σ} isn't empty. Supposing in (12) $z=1,\ \beta=\omega u^{\sigma}$ we get

$$\int_{D'_\Sigma} \mathrm{M}\left(\omega u^\sigma\right) = \sigma \int_{\partial D_\Sigma^+} \left(\omega_v u^\sigma + \sigma u^{\sigma-1} \frac{\partial u}{\partial v} \right) ds \le$$

$$\le M^\sigma \int_{\partial D_\Sigma^+} \omega ds + \sigma M^{\sigma-1} \int_{\partial D_\Sigma^+} \left| \frac{\partial u}{\partial v} \right| ds \le C_5\left(a_0, M, \sigma, C_1\right) \varepsilon$$

But, on the other hand

$$\mathrm{M}\left(u^\sigma\right) = \sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij} \frac{\partial \omega u^\sigma}{\partial x_i} \right) =$$

$$= \sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij}\omega\left(\sigma u^{\sigma-1} \frac{\partial u}{\partial x_j} \right) + \sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij}\omega_{x_i} \frac{\partial u^\sigma}{\partial x_j} \right)\right) =$$

$$= \sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij}\omega\sigma u^{\sigma-1} \frac{\partial u}{\partial x_j} \right) + \sum_{i,j=1}^n \frac{\partial}{\partial x_i}\left(a_{ij}\sigma u^{\sigma-1}\omega x \frac{\partial u}{\partial x_j} \right) =$$

$$= \sigma\omega u^{\sigma-1}\mathrm{M}\left(u\right) + \sigma\omega \frac{\partial}{\partial x_i}\left(a_{ij}\, u^{\sigma-1} \frac{\partial u}{\partial x_j} \right) + \sigma u^{\sigma-1} \frac{\partial}{\partial x_i}\left(a_{ij}\omega \frac{\partial u}{\partial x_j} \right) + \beta =$$

$$= \sigma\omega u^{\sigma-1}\mathrm{M}\left(u\right) + \sigma\omega u^{\sigma-1} \frac{\partial}{\partial x_i}\left(a_{ij} \frac{\partial u}{\partial x_j} \right) + \sigma\omega a_{ij} u_{x_j} \left(\sigma-1\right) u^{\sigma-2} u_{x_i} +$$

$$= \sigma u^{\sigma-1}\omega_{x_i}\left(a_{ij} \frac{\partial u}{\partial x_j} \right) + \sigma u^{\sigma-1}\omega \frac{\partial}{\partial x_i}\left(a_{ij} \frac{\partial u}{\partial x_j} \right) + \beta =$$

$$= 3\sigma\omega u^{\sigma-1}\mathrm{M}\left(u\right) + \sigma\left(\sigma-1\right) a_{ij}\, u x_i u_{x_i} u^{\sigma-2}\omega + \sigma u^{\sigma-1} u_{x_i} a_{ij} u_{x_j} + \beta =$$

$$= \sigma \int_{D_\Sigma^+} d_i\left(x\right) u_{x_i} u\omega\, dx - \sigma\left(\sigma-1\right) \int_{D_\Sigma^+} u^\sigma \omega\left(x\right) c\left(x\right) dx +$$

$$+\sigma\left(\sigma-1\right) \int_{D_\Sigma^+} \sum_{i,j=1}^n u^{\sigma-2}\omega\left(x\right) a_{ij}\, u_i u_j dx +$$

$$+\left(2u+1\right) \int_{D_\Sigma^+} \sum_{i,j=1}^n a_{ij}\, u_j \omega_{x_j} u^{\sigma-1}$$

Hence, we conclude

$$\sigma\left(\sigma-1\right) \int_{D_\Sigma^+} \omega^2 u^{\sigma-2} \left|\nabla u\right|^2 dx \le$$

$$\le d_0 \int_{D_\Sigma^+} u^{\sigma-1}\omega u_i dx \le d_0 \int_{D_\Sigma^+} u^{\sigma-1}\omega u_i dx \le \frac{d_0 \varepsilon}{2} \int_{D_\Sigma^+} u^\sigma dx \quad (15)$$

Let $D^+ = \left\{x : x \in D, u\left(x\right) > 0\right\}$, D_1^+ D^+ be an arbitrary connected component of D^+. Subject to the arbitrariness of ε from (15) we get

$$\left(\sigma-1\right)\gamma\int_{D_1^+}\omega u^{\sigma-2}\left|\nabla u\right|^2dx\le d_0\int_{D_1^+}\omega u^{\sigma-1}\sum_{i=1}^{n}\left|u_i\right|dx.$$

Thus, for any $\mu>0$

$$\left(\sigma-1\right)\gamma\int_{D_1^+}\omega u^{\sigma-2}\left|\nabla u\right|^2dx\le\frac{d_0\varepsilon}{2}\int_{D_1^+}\omega u^{\sigma-2}\left(\sum_{i=1}^{n}\left|u_i\right|\right)^2dx+\frac{d_0}{2\mu}\int_{D_1^+}\omega u^{\sigma}dx\le$$

$$\le\frac{d_0\mu n}{2}\int_{D_1^+}\omega u^{\sigma-2}\left|\nabla u\right|^2dx+\frac{d_0}{2\mu}\int_{D_1^+}\omega u^{\sigma}dx \qquad (16)$$

But, on the other hand

$$I=-\sigma\sum_{i=1}^{1}\int_{D_1^+}x_1\omega u^{\sigma-1}u_idx=-\sum_{i=1}^{n}\int_{D_1^+}x_1\omega\left(u^{\sigma}\right)_idx=n\int_{D_1^+}\omega u^{\sigma}dx\ ,$$

and besides, for any $\beta>0$

$$I=\frac{\sigma\beta}{2}\int_{D_1^+}r^2\omega u^{\sigma}dx+\frac{\sigma}{2\beta}\int_{D_1^+}u^{\sigma-2}\omega^2\left|\nabla u\right|^2dx$$

Then

$$I\le\frac{\sigma\beta}{2}\int_{D_1^+}r^2\omega u^{\sigma}dx+\frac{\sigma}{2\beta}\int_{D_1^+}\omega^2\left|\nabla u\right|^2u^{\sigma-2}dx,$$

where $r=\left|x\right|$. Denote by $k\left(D\right)$ the quantity $\sup\limits_{x\in D}\left|x\right|$. Without loss of generality, we'll suppose, that $k\left(D\right)=1$. Then

$$I=\frac{\sigma}{2\beta}\int_{D_1^+}\omega u^{\sigma}dx+\frac{\sigma}{2\beta}\int_{D_1^+}\omega^2u^{\sigma-2}\left|\nabla u\right|^2dx$$

Thus,

$$\left(n-\frac{\sigma\beta}{2}\right)\int_{D_1^+}\omega u^{\sigma}dx+\frac{\sigma}{2\beta}\int_{D_1^+}\omega^2u^{\sigma-2}\left|\nabla u\right|^2dx$$

Now, choosing $\beta=\dfrac{n}{\sigma}$ we finnaly obtain

$$\int_{D_1^+}\omega u^{\sigma}dx\le\frac{\sigma^2}{n^2}\int_{D_1^+}\omega^2u^{\sigma-2}\left|\nabla u\right|^2dx \qquad (17)$$

Subject to (17) in (16), we conclude

$$\left(\sigma-1\right)\gamma\int_{D_1^+}\omega^2u^{\sigma-2}\left|\nabla u\right|^2dx\le\frac{d_0\varepsilon n}{2}+\frac{d_0\sigma^2}{2\varepsilon n^2}\int_{D_1^+}\omega^2u^{\sigma-2}\left|\nabla u\right|^2dx \qquad (18)$$

Now choose μ such that

$$\left(\sigma-1\right)\gamma>\frac{d_0\mu n}{2}+\frac{d_0\sigma^2}{2\mu n^2} \qquad (19)$$

Then from (17)-(19) it will follow that $u(x)\equiv0$ in D_1^+, and thus $u(x)\equiv0$

in D. Suppose, that $\mu=\dfrac{(\sigma-1)\gamma}{d_0 n}$ .Then (19) is equivalent to the condition

$$n>\left(\frac{\sigma}{\sigma-1}\right)^2\left(\frac{d_0}{\gamma}\right)^2 \tag{20}$$

At first, suppose, that

$$n>\left(\frac{d_0}{\gamma}\right)^2 \tag{21}$$

Let's choose and fix such big $\sigma \geq 2$, that by fulfilling (21) the inequality (20) was true. Thus the theorem is proved, if with respect to the condition (21) is fulfilled. Show, that it is true for any $n \geq 3$. For this, at first, note, that if $k(D) \neq 1$, then condition (21) will take the form

$$n>\left(\frac{d_0 k(D)}{\gamma}\right)^2$$

Now, let the condition (21) be not fulfilled. Denote by k the least natural number, for which

$$n+k>\left(\frac{d_0}{\gamma}\right)^2 \tag{22}$$

Consider $(n+k)$ dimensional semi-cylinder $D'=D\times(-\delta_0,\delta_0)\times\ldots\times,\times(-\delta_0,\delta)$, where the number $\delta_0>0$ will be chosen later. Since, $\omega(D)=1$ then $\omega(D')\leq 1+\delta_0\sqrt{k}$. Let's choose and $fix\delta_0$ such small, that along with the condition (22) the condition

$$n+k>\left(\frac{d_0\omega(D')}{\gamma}\right)^2 \tag{23}$$

was fulfilled too.

Let

$$y=(x_1,\ldots,x_n,x_{n+1},\ldots,x_{n+k}),\ \ E'=E\times\underbrace{[-\delta_0,\delta_0]\times\ldots\times[-\delta_0,\delta_0]}_{k\ times}$$

Consider on the domain D' the equation

$$\mathsf{L}'_{\vartheta}=\sum_{i,j=1}^{n}a_{ij}(x)\vartheta_{ij}+\sum_{i=1}^{n}\frac{\partial^2\vartheta}{\partial x_{n+i}^2}+\sum_{i=1}^{n}b_i(x)\vartheta_i+c(x)\vartheta=0 \tag{24}$$

It is easy to see, that the function $\vartheta(y)=u(x)$ is a solution of the equation (24) in $D'\setminus E'$. Besides, $m_H^{n+k-2+\lambda}(E')=(2\delta_0)^k\, m_H^{n-2+\lambda}(E)=0$, the function $\vartheta(y)$

vanishes on $\left(\partial D \times \underbrace{\left[-\delta_0,\delta_0\right] \times \ldots \times \left[-\delta_0,\delta_0\right]}_{k\ times} \right) \setminus E'$ and $\dfrac{\partial \vartheta}{\partial v'} = 0 \qquad x_{n+i} = \pm\delta_0,\ \ i = 1,\ldots,k$

where $\dfrac{\partial}{\partial v^1}$ is a derivative by the conormal, generated by the operator $L^{'}$. Noting, that $\gamma\left(\mathsf{L}'\right) = \gamma\left(\mathsf{L}\right)$, $d_0\left(\mathsf{L}'\right) = d_0\left(\mathsf{L}\right)$ and subject to the condition (23), from the proved above we conclude, that $\vartheta(y) \equiv 0$, i.e. $D^{'}$. The theorem is proved.

Remark. As is seen from the proof, the assertion of the theorem remains valid, if instead of the condition (2) it is required, that the coefficients $a_{ij}\left(x\right)\left(i,j=1,\ldots,n\right)$ have to satisfy in domain D the uniform Lipschitz condition.

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AŞAĞI TƏRTİBLİ HƏDLƏRƏ MALİK CIRLAŞAN ELLİPTİK TƏNLİYİN HƏLLƏRİNİN ARADAN QALDIRILA BİLƏN ÇOXLUQLARI

O.S.ƏLİYEV

XÜLASƏ

Məqalədə aşağı tərtibli hədlərə malik cırlaşan divergent olmayan ikinci tərtib elliptik tənliklərə baxılır. Çəkili Hölder fəzalarında bu tənliklərə nəzərən kompaktın aradan qaldırılması üçün kafi şərtlər alınmışdır.

Açar sözlər: aradan qaldırılan, həll, cırlaşan, tənlik.

ОБ УСТРАНИМЫХ МНОЖЕСТВАХ ВЫРОЖДАЮЩИХСЯ ЭЛЛИПТИЧЕСКИХ УРАВНЕНИЙ С ЧЛЕНАМИ МЕНЬШЕГО ПОРЯДКА

О.С.АЛИЕВ

РЕЗЮМЕ

В статье рассматриваются недивергентные вырождающиеся эллиптические уравнения второго порядка. Получены достаточные условия для устранимости компакта относительного этих уравнений в весовых пространствах Гельдера.

Ключевые слова: устранимые особенности, решение, вырождающийся, уравнение.

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