ON REMOVABLE SETS OF SOLUTIONS OF DEGENERATE ELLIPTIC EQUATIONS WITH MINOR TERMS

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In this paper we consider the nondivergent degenerate elliptic equations of the second order with minor terms. The sufficient condition of removability of compact with respect to these equations in the weight space of Hölder functions was found.

Key words: removable, solution, degenerate, equation

The aim of the given paper is finding sufficient condition of removability of compact with respect to the equation in the space \( C^i_\lambda(D) \). This problem has been investigated by many degenerate elliptic researchers. For the Laplace equation the corresponding result was found by L. Carleson [1]. Concerning the second order elliptic equations of divergent structure, we show in this direction the papers [2], [3]. For a class of non-divergent elliptic equations of the second order with discontinuous coefficients of removability condition for a compact in the space \( C^i(D) \) was found in [4]. Mention also papers [5-7], in which the conditions of removability for a compact in the space of continuous functions have been obtained. The removable sets of solutions of the second order elliptic and parabolic equations in nondivergent form was considered in [10]-[12]. In [13], T. Kilpelainen and X. Zhong have studied the divergent quasilinear equation without minor members proved the removability of compact. Removable sets for pointwise solutions of elliptic partial differential equations was found by J. Diederich [14]. Removable singularities of solutions of linear partial differential equations was considered in R. Harvey, J. Polking [15]. On removable sets at the boundary for subharmonic functions has been investigated by B. Dahlberg [16].

Let \( D \) be a bounded domain situated in \( n \) dimensional Euclidean space \( E_n \) of points \( x = (x_1, ..., x_n), \ n \geq 3, \ \partial D \) be its boundary. Consider in \( D \) the following elliptic equation
\[ L_u = \sum_{i,j=1}^{n} a_{ij}(x)u_{ij} + \sum_{i=1}^{n} b_{i}(x)u_{i} + c(x)u = 0 \]  
(1)

in supposition that \( \|a_{ij}(x)\| \) is a real symmetric matrix, moreover \( \omega(x) \) be a positive measurable function satisfying the doubling condition: for concentric balls \( B_R^x \) or \( R \) and \( 2R \) radius, there exists such a constant \( \gamma \)

\[ \omega(B_R^x) \geq \gamma \omega(B_{2R}^x), \]

where for the measurable sets \( E \omega(E) \) means \( \omega(y)dy \)

\[ \gamma \int_{E}^{\infty} \omega(x) \leq \sum_{i,j=1}^{n} a_{ij}(x)\xi_{i}\xi_{j} \leq \gamma^{-1} \omega(x) \int_{E}^{\infty} \xi_{i}^{2}; \quad \xi \in E, \quad x \in D, \]  
(2)

\[ a_{i}(x) \in C_{a}^{1}(D); \quad i,j,1,...,n, \]  
(3)

\[ |b_{i}(x) \leq b_{0}; \quad -b_{0} \leq c(x) \leq 0; \quad i = 1,...,n; \quad x \in D. \]  
(4)

Here \( u_{ij} = \frac{\partial u}{\partial x_j}, \quad u_{yj} = \frac{\partial^2 u}{\partial x_i \partial x_j}; \quad i,j = 1,...,n; \quad \gamma \in (0,1] \) and \( b_{0} \geq 0 \) are constants.

Besides we'll suppose that the lower coefficients of the operator \( L \) are measurable functions in \( D \). Let \( \lambda \in (0,1) \) be a number. Denote by \( C_{\omega}^{1,\lambda}(D) \) a Banach space of the functions \( u(x) \) defined in \( D \) with the finite norm.

\[ \|u\|_{C_{\omega}^{1,\lambda}(D)} = \sup_{x \in D} \omega(x) \bigg| u(x) + \sup_{x,y \in D} \frac{|u(x) - u(y)|}{|x - y|^{\lambda}} \bigg|. \]

The compact \( E \subset \overline{D} \) is called removable with respect to the equation (1) in the space \( C_{\omega}^{1,\lambda}(D) \) if from

\[ L_u = 0, \quad x \in D \setminus E; \quad u \bigg|_{\partial D \cup E} = 0; \quad u(x) \in C_{\omega}^{1,\lambda}(D) \]  
(5)

it follows that \( u(x) \equiv 0 \) in \( D \).

Denote by \( B_R(z) \) and \( S_R(z) \) the ball \( \{x : |x - z| < R\} \) and the sphere \( \{x : |x - z| = R\} \) of radius \( R \) with the center at the point \( z \in E_n \) respectively.

We'll need the following generalization of mean value theorem belonging to E.M. Landis and M.L. Gerver [8] in weight case.

Lemma. Let the domain \( G \) be situated between the spheres \( S_R(0) \) and \( S_{2R}(0) \), moreover the intersection \( \partial G \cap \{x : R < |x| < 2R\} \) be a smooth surface. Further, let in \( G \) the uniformly positive determined matrix \( \|a_{ij}(x)\|; \quad i,j = 1,...,n \)
and the function \( u(x) \in C^\omega(G) \cap C^\omega(\overline{G}) \) be given. Then there exists the piecewise smooth surface \( \Sigma \) dividing in \( G \) the spheres \( S_{\varrho}(0) \) and \( S_{2\varrho}(0) \) such that
\[
\int_\Sigma \left| \frac{\partial u}{\partial \nu} \right| ds \leq K_{osc} u \frac{\omega(G)}{R^2}.
\]

Here \( K > 0 \) is a constant, depending only on matrix \( \|a_{i\varrho}(x)\| \) and \( n \), and \( \frac{\partial u}{\partial \nu} \) is a derivative by conormals determined by the equality
\[
\frac{\partial u(x)}{\partial \nu} = \sum_{i,j=1}^n a_{ij}(x) \frac{\partial u(x)}{\partial x_j} \cos(n,x_j)^{-1/2},
\]
where \( \cos(n,x_j); j = 1,...,n \)-are directing cosine of unit external normal vector to \( \Sigma \).

Denote by \( W^1_{2,0}(D) \) the Banach space of the functions \( u(x) \) defined in \( D \) with the finite norm
\[
\|u\|_{W^1_{2,0}(D)} = \left( \int_D \left( \omega(x)^2 + \sum_{i=1}^n u_i^2 \right) dx \right)^{1/2}
\]
and let \( W^1_{2,0}(D) \) be a completion \( C^{0,\omega}_0(D) \) by the norm of the space \( W^1_{2,0}(D) \).

By \( m^s_{\mu}(A) \) we'll denote the Hausdorff measure of the set \( A \) of order \( s > 0 \). Further everywhere the notation \( C(\ldots) \) means, that the positive constant \( C \) depends only on content of brackets.

Theorem 1. Let \( D \) be a bounded domain in \( E_n \), \( E \subset \overline{D} \) be a compact. If with respect to the coefficients of the operator \( L \) the conditions (1)-(3) are fulfilled, then for removability of the compact \( E \) with respect to the equation (5) in the space \( C^{1,\omega}_0(D) \) it sufficies that
\[
m^{s-2,1}_{\mu}(E) = 0
\]

Proof. At first we show that without loss of generality we can suppose the condition \( \partial D \in C^1 \) to be fulfilled. Suppose, that the condition (6) provides the removability of the compact \( E \) for the domains, whose boundary is the surface of the class \( C^1 \), but \( \partial D \in C^1 \) and by fulfilling (6) the compact \( E \) is not removable. Then the problem (5) has non-trivial solution \( u(x) \), moreover \( u|_E = f(x) \) and \( f(x) \neq 0 \). We always can suppose the lowest coefficients of the operator \( L \) to be infinitely differentiable in \( D \). Moreover, without loss of generality, we'll suppose that the coefficients of the operator \( L \) are extended to a ball \( B \supset \overline{D} \) with saving the conditions (2)-(4). Let \( f^+(x) = \max\{f(x),0\} \),

70
$f^-(x) = \min\{f(x), 0\}$ and $u^+(x)$ be generalized by Wiener (see [8]) solutions of the boundary value problems

$$L_u^\pm = 0, \ x \in D \setminus E; \quad u^\pm \bigg|_{\partial D \setminus E} = 0; \quad u^\pm \bigg|_E = f^\pm.$$  

Evidently, by $u(x) = u^+(x) + u^-(x)$. Further, let $D'$-be such a domain, that

$$\partial D' \in C^1, \quad D \subset \overline{D'}, \quad D' \subset B,$$

and $v^\pm(x)$ be solutions of the problems

$$Lv^\pm = 0, \ x \in D' \setminus E; \quad v^\pm \bigg|_{\partial D'} = 0; \quad v^\pm \bigg|_E = f^\pm; \quad v^i(x) \in C^\omega(D').$$

By the maximum principle for $x \in D$

$$0 \leq u^+(x) \leq v^+(x), \quad v^-(x) \leq u^-(x) \leq 0.$$  

But according to our supposition $v^+(x) \equiv v^-(x) \equiv 0$. Hence, it follows, that $u(x) \equiv 0$. So, we'll suppose, that $\partial D \in C^1$. Now, let $u(x)$ be a solution of the problem (5), and the condition (6) be fulfilled. Give an arbitrary $\varepsilon > 0$. Then there exists a sufficiently small positive number $\delta$ and a system of the balls

$$\{B_{\varepsilon}(x^\delta)\}, \ k = 1, 2, \ldots, \text{such that } r_k < \delta, \ E \subset \bigcup_{k=1}^\infty B_{\varepsilon}(x^\delta) \text{ and }$$

$$\sum_{k=1}^\infty r_k^{n-2+\lambda} < \varepsilon. \quad (7)$$

Consider a system of the spheres $\{B_{2r_k}(x^\delta)\}$, and let $D_k = D \cap B_{2r_k}(x^\delta), k = 1, 2, \ldots,$. Without loss of generality we can suppose, that the cover $\{B_{2r_k}(x^\delta)\}$ has a finite multiplicity $a_0(n)$. By the Landis-Gerver theorem for every $k$ there exists a piece-wise smooth surface $\Sigma_k$ dividing in $D_k$ the spheres $S_{r_k}(x^\delta)$ and $S_{2r_k}(x^\delta)$, such that

$$\int_{\Sigma_k} \frac{\partial u}{\partial v} ds \leq \text{osc}u \frac{\omega(D_k)}{r_k^2}. \quad (8)$$

Since $u(x) \in C^\omega(D)$, there exists a constant $H_1 > 0$, depending only on the function $u(x)$, such that

$$\text{osc}u \omega \leq H_1 (2r_k)^{-1}\quad (9)$$

Besides

$$\omega(D_k) \leq \text{mes}_{r_k} B_{2r_k}(x^\delta) = \Omega_n 2^n r_k^n; \quad k = 1, 2, \ldots, \quad (10)$$

where $\Omega_n = \text{mes}_{r_k} B_1(0)$. Using (9)-(10) in (8) we get

$$\int_{\Sigma_k} \frac{\partial u}{\partial v} ds \leq \frac{C_1 r_k^{n+\lambda}}{r_k^n}; \quad k = 1, 2, \ldots, \quad (11)$$

where $C_1 = KH_1 2^{n+\lambda}$.  

71
Let \( D_\Sigma \) be an open set situated in \( D \setminus E \), whose boundary consists of unification of \( \Sigma \) and \( \Gamma \) where \( \Sigma = \bigcup_{k=1}^n \Sigma_k \), \( \Gamma = \partial D \setminus \bigcup_{k=1}^n D_k^+ \). \( D_k^+ \) is a part of \( D_k \) remaining after the removing of points situated between \( \Sigma \) and \( S_{2k}(x^k); \ k = 1,2,\ldots \). Denote by \( D_\Sigma^'l \) the arbitrary connected component \( D_\Sigma \), and by \( M \) we denote the ellipitic operator of divergent structure

\[
M = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left( a_{ij}(x) \frac{\partial}{\partial x_j} \right).
\]

According to Green formula for any functions \( z(x) \) and \( W(x) \) pertaining to the intersection \( C^2(D_\Sigma^' \cap C(D_\Sigma^') \) we have

\[
\int_{\partial D_\Sigma^'} \left( z M \beta - \beta M z \right) dx = \int_{\partial D_\Sigma} \left( z \frac{\partial \beta}{\partial v} - \beta \frac{\partial z}{\partial v} \right) ds. \tag{12}
\]

As is known \( \partial D \in C^1 \) then \( u(x) \in C^1 \left( D_\Sigma^' \right) \cap C^1 \left( \overline{D_\Sigma}^' \right) \) (see [9]). From (12) choosing the functions \( z = 1, \beta = \omega u^2 \) we have

\[
\int_{\partial D_\Sigma} M \left( \omega u^2 \right) dx = 2 \int_{\partial D_\Sigma} \omega \frac{\partial u}{\partial v} ds + \int_{\partial D_\Sigma} \omega u^2 ds.
\]

But \( |u(x)| \leq M < \infty \) for \( x \in \overline{D} \). Let's put the condition:

\[
\omega_{x_i} < c\omega \quad (*)
\]

By virtue of condition (*) and \( \int_{\partial D_\Sigma} \omega u^2 ds < C_3 M_\epsilon \), subject to (11) and (7) we conclude

\[
\int_{\partial D_\Sigma} M \left( \omega u^2 \right) dx \leq 2M_\omega \sum_{k=1}^n \int_{\partial D_\Sigma} \omega \frac{\partial u}{\partial v} ds + \int_{\partial D_\Sigma} \omega u^2 dx \leq 
\]

\[
\leq 2M_\omega \omega C_1 \sum_{k=1}^n r_k^{n-2+\epsilon} + \epsilon Mc_2 < C_3 \epsilon, \tag{13}
\]

w \( C_3 = 2M_\omega C_1 \) here.

On the other side

\[
M \left( \omega u^2 \right) = 6u\omega M(u) + \\
+2 \sum_{i,j=1}^n \omega a_{ij} u_i u_j + (2u+1) \sum_{i,j=1}^n a_{ij} u_i \omega u_j + \sum_{i,j=1}^n \frac{\partial a_{ij}}{\partial x_j} u \omega u_j + \sum_{i,j=1}^n a_{ij} \omega u_j u_j
\]

and besides,

\[
M_u = L_u + \sum_{i=1}^n d_i(x) u_j - c(x) u,
\]

where
\[ d_i(x) = \sum_{j=1}^{n} \frac{\partial a_{ij}(x)}{\partial x_j} - b_i(x), \quad i = 1, \ldots, n. \]

It is evident, that by virtue of conditions (2)-(3) \(|d_i(x)| \leq d_0 < \infty; \quad i = 1, \ldots, n.\)

Thus from (13) we obtain
\[
6 \int_{\mathbb{E}} u_{\omega} \sum_{i=1}^{n} d_i(x) u_i dx - 6 \int_{\mathbb{E}} u^{\omega} c(x) dx + 2 \int_{\mathbb{E}} \sum_{i,j=1}^{n} a_{ij} u_i u_j dx +
\]
\[
+ (2u + 1) \int_{\mathbb{E}} \sum_{i,j=1}^{n} a_{ij} u_i \omega_j dx + \int_{\mathbb{E}} \sum_{i,j=1}^{n} \frac{\partial a_{ij}}{\partial x_j} u \omega_j dx + |\nabla u|^2 dx +
\]
\[
+ \int \sum_{i,j=1}^{n} a_{ij} u \omega_j dx < C_{3e}
\]

Hence for any \( \alpha > 0 \) it follows that
\[
2\gamma \int_{\mathbb{E}} |\nabla u|^2 dx < 6d_0 \int_{\mathbb{E}} \omega |u_i| dx + 6 \int_{\mathbb{E}} u^{\omega} c(x) dx + (2u + 1) \int_{\mathbb{E}} a_{ij} u_i \omega_j dx +
\]
\[
+ d_0 \int_{\mathbb{E}} u \omega_j dx + \int_{\mathbb{E}} a_{ij} u \omega_j + C_{3e} \leq \frac{d_0}{\varepsilon} \int_{\mathbb{E}} |u|^2 dx + \frac{d_0}{\varepsilon} \int_{\mathbb{E}} \omega |\nabla u|^2 dx +
\]
\[
+ (2n + 1) \int_{\mathbb{E}} u_j \omega dx + d_0 \int_{\mathbb{E}} u \omega dx + \gamma C_{3e} \leq \frac{d_0}{\varepsilon} M\text{mes}_s D +
\]
\[
+ \frac{(2M + 1)\gamma}{\varepsilon} \text{mes}_s D + d_0 M \omega(D) + \gamma C_4 M \omega(D) + C_3 \varepsilon
\]

If we'll take into account that
\[
|\omega_{j+j}| < C_4 \omega(x),
\]

then from here we have that
\[
\int_{\mathbb{E}} \omega_j |\nabla u|^2 dx \leq C_3
\]

where \( C_3 = (6d_0 + (2M + 1)) M\text{mes}_s D + (d_0 M + \gamma C_4 M) \omega(D) + \frac{C_3}{\gamma} \). Without loss of generality we assume that \( \varepsilon \leq 1 \). Hence we have
\[
\int_{\mathbb{E}} \omega_j |\nabla u|^2 dx \leq C_6
\]

Thus \( u(x) \in W^{1,2}_\omega(D) \). From the boundary condition and \( \text{mes}_n (\partial D \cap E) = 0 \) we get \( u(x) \in W^{1,2}_\omega(D) \). Now, let \( \sigma \) be a number, which will be chosen later, \( D^+_{\Sigma} = \{ x : x \in D_{\Sigma}, u(x) > 0 \} \). Without loss of generality, we suppose, that the set \( D^+_{\Sigma} \) isn't empty. Supposing in (12) \( z = 1, \beta = \omega u^\sigma \) we get

73
\[
\int_{\nu_x^*} M (\omega u^*) = \sigma \int_{\nu_x^*} \left( \omega u^* + \sigma u^{*-1} \frac{\partial u}{\partial v} \right) ds \leq M^* \int_{\nu_x^*} \omega ds + \sigma M^{*-1} \int_{\nu_x^*} \left| \frac{\partial u}{\partial v} \right| ds \leq C, (a, M, \sigma, C) e
\]

But, on the other hand
\[
M (u^*) = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left( a_{ij} \frac{\partial \omega u^*}{\partial x_j} \right) = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left( a_{ij} \omega \sigma u^{*-1} \frac{\partial u}{\partial x_j} \right) + \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left( a_{ij} \omega \frac{\partial u}{\partial x_j} \right) + \beta = \sigma \omega u^{*-1} M (u) + \sigma \omega u^{*-1} \frac{\partial}{\partial x_i} \left( a_{ij} \frac{\partial u}{\partial x_j} \right) + \sigma u^{*-1} \frac{\partial}{\partial x_i} \left( a_{ij} \omega \frac{\partial u}{\partial x_j} \right) + \beta = 3 \sigma \omega u^{*-1} M (u) + \sigma (\sigma - 1) a_{ij} u_{ij} u^{*-2} \omega + \sigma u^{*-1} u_{ij} a_{ij} u_{ij} + \beta = \sigma \int_{\nu_x^*} d_i (x) u_{ij} u_{ij} dx - \sigma (\sigma - 1) \int_{\nu_x^*} u^\omega (x) c(x) dx + \sigma (\sigma - 1) \int_{\nu_x^*} \sum_{i,j=1}^n u^{*-2} \omega (x) a_{ij} u_{ij} dx + (2u + 1) \int_{\nu_x^*} \sum_{i,j=1}^n a_{ij} u_{ij} u^{*-1}
\]

Hence, we conclude
\[
\sigma (\sigma - 1) \int_{\nu_x^*} \omega u^{*-2} |\nabla u|^2 dx \leq d_1 \int_{\nu_x^*} u^{*-1} \omega u dx \leq d_1 \int_{\nu_x^*} u^{*-1} \omega u dx \leq d_1 e \int_{\nu_x^*} u dx \quad (15)
\]

Let \( D^+ = \{ x : x \in D, u(x) > 0 \} \), \( D^- \) be an arbitrary connected component of \( D^+ \). Subject to the arbitrariness of \( \varepsilon \) from (15) we get
Thus, for any \( \mu > 0 \)

\[
(\sigma - 1) \gamma \int_{\Omega_i} \omega u^{\sigma - 2} |\nabla u|^2 \, dx \leq d_{\omega} \int \omega u^{\sigma - 2} \left( \sum_{i=1}^n u_i \right)^2 \, dx + \frac{d_{\omega}}{2\mu} \int \omega u^\sigma \, dx \\
\leq \frac{d_{\omega} \mu n}{2} \int \omega u^{\sigma - 2} |\nabla u|^2 \, dx + \frac{d_{\omega}}{2\mu} \int \omega u^\sigma \, dx
\]  

(16)

But, on the other hand

\[
I = -\sigma \sum_{i=1}^n \int x_i \omega u^{\sigma - 1} u_i \, dx = -\sum_{i=1}^n \int x_i \omega \left( u^\sigma \right) \, dx = n \int \omega u^\sigma \, dx
\]

and besides, for any \( \beta > 0 \)

\[
I = \frac{\sigma \beta}{2} \int_{\Omega_i} \omega^2 u^\sigma \, dx + \frac{\sigma}{2\beta} \int u^{\sigma - 2} \omega^2 |\nabla u|^2 \, dx
\]

Then

\[
I \leq \frac{\sigma \beta}{2} \int_{\Omega_i} \omega^2 u^\sigma \, dx + \frac{\sigma}{2\beta} \int u^{\sigma - 2} \omega^2 |\nabla u|^2 \, dx,
\]

where \( r = |\chi| \). Denote by \( k(D) \) the quantity \( \sup_{x \in D} |\chi| \). Without loss of generality, we'll suppose, that \( k(D) = 1 \). Then

\[
I = \frac{\sigma}{2\beta} \int \omega u^\sigma \, dx + \frac{\sigma}{2\beta} \int u^{\sigma - 2} \omega^2 |\nabla u|^2 \, dx
\]

Thus,

\[
\left( n - \frac{\sigma \beta}{2} \right) \int \omega u^\sigma \, dx + \frac{\sigma}{2\beta} \int u^{\sigma - 2} \omega^2 |\nabla u|^2 \, dx
\]

Now, choosing \( \beta = \frac{n}{\sigma} \) we finnaly obtain

\[
\int \omega u^\sigma \, dx \leq \frac{\sigma^2}{n^2} \int \omega^2 u^{\sigma - 2} |\nabla u|^2 \, dx
\]

(17)

Subject to (17) in (16), we conclude

\[
(\sigma - 1) \gamma \int_{\Omega_i} \omega u^{\sigma - 2} |\nabla u|^2 \, dx \leq \frac{d_{\omega} \epsilon n}{2} + \frac{d_{\omega} \sigma^2}{2\epsilon n^2} \int \omega u^{\sigma - 2} |\nabla u|^2 \, dx
\]

(18)

Now choose \( \mu \) such that

\[
(\sigma - 1) \gamma > \frac{d_{\omega} \mu n}{2} + \frac{d_{\omega} \sigma^2}{2\mu n^2}
\]

(19)

Then from (17)-(19) it will follow that \( u(x) \equiv 0 \) in \( D_1^\ast \), and thus \( u(x) \equiv 0 \).
in $D$. Suppose, that $\mu = \frac{(\sigma - 1)\gamma}{d_0 n}$. Then (19) is equivalent to the condition

$$n > \left(\frac{\sigma}{\sigma - 1}\right)^2 \left(\frac{d_0}{\gamma}\right)^2$$  \tag{20}

At first, suppose, that

$$n > \left(\frac{d_0}{\gamma}\right)^2$$  \tag{21}

Let's choose and fix such big $\sigma \geq 2$, that by fulfilling (21) the inequality (20) was true. Thus the theorem is proved, if with respect to the condition (21) is fulfilled. Show, that it is true for any $n \geq 3$. For this, at first, note, that if $k(D) \neq 1$, then condition (21) will take the form

$$n > \left(\frac{d_0 k(D)}{\gamma}\right)^2$$

Now, let the condition (21) be not fulfilled. Denote by $k$ the least natural number, for which

$$n + k > \left(\frac{d_0}{\gamma}\right)^2$$  \tag{22}

Consider $(n + k)$ dimensional semi-cylinder $D' = D \times (-\delta_0, \delta_0) \times ... \times (-\delta_0, \delta_0)$, where the number $\delta_0 > 0$ will be chosen later. Since, $\omega(D) = 1$ then $\omega(D') \leq 1 + \delta_0 \sqrt{k}$. Let's choose and fix $\delta_0$ such small, that along with the condition (22) the condition

$$n + k > \left(\frac{d_0 \omega(D')}{\gamma}\right)^2$$  \tag{23}

was fulfilled too.

Let

$$y = (x_1, ..., x_n, x_{n+1}, ..., x_{n+k}), \quad E' = E \times [-\delta_0, \delta_0] \times ... \times [-\delta_0, \delta_0]$$

Consider on the domain $D'$ the equation

$$L'_{\vartheta} = \sum_{i,j=1}^n a_{ij}(x) \vartheta_{ij} + \sum_{i=1}^n \frac{\partial^2}{\partial x_{n+i}^2} \vartheta + \sum_{i=1}^n b_i(x) \vartheta + c(x) \vartheta = 0$$  \tag{24}

It is easy to see, that the function $\vartheta(y) = u(x)$ is a solution of the equation (24) in $D \setminus E'$. Besides, $m_{n-k-2+\delta}^{n-k-2+\delta}(E') = (2\delta_0)^k m_{n-k-2+\delta}^{n-k-2+\delta}(E) = 0$, the function $\vartheta(y)$
vanishes on \[ \partial D \times \left[ \partial \sigma_1, \partial \sigma_2 \times \ldots \times \left[ \partial \sigma_i, \partial \sigma_j \right] \right] \backslash E' \] and \[ \frac{\partial \beta}{\partial v^i} = 0 \quad x_{\alpha i} = \pm \delta_{\alpha i}, \quad i = 1, \ldots, k \]

where \( \frac{\partial}{\partial v^i} \) is a derivative by the conormal, generated by the operator \( L' \). Noting, that \( \gamma (L') = \gamma (L) \), \( d_\sigma (L') = d_\sigma (L) \) and subject to the condition (23), from the proved above we conclude, that \( \beta (y) \equiv 0 \), i.e. \( D' \). The theorem is proved.

Remark. As is seen from the proof, the assertion of the theorem remains valid, if instead of the condition (2) it is required, that the coefficients \( a_{ij} (x) (i, j = 1, \ldots, n) \) have to satisfy in domain \( D \) the uniform Lipschitz condition.

REFERENCES


АСАĞI TƏRTİBLİ HƏDLƏRƏ MALİK ÇIRĻAŞAN ELLİPTİK TƏNLİYİN HƏLLƏRİNİN ARADAN QALDIRILA BİLƏN ÇOXLUQLARI

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XÜLASƏ

Maqalədə aşağı tərtibli hədlərə malik çirlaşan divergent olmayan ikinci tərtib elliptik tənliklərə baxılır. ÇəkiHölder fazalarında bu tənliklərə nəzərən kompaktn aradan qaldırıl-masti üçün kəfi şərtlər alınmışdır.

Açar səzər: aradan qaldırılan, həll, çirlaşan, tənlik.

ОБ УСТРАНИМЫХ МНОЖЕСТВАХ ВЫРОЖДАЮЩИХСЯ ЭЛЛИПТИЧЕСКИХ УРАВНЕНИЙ С ЧЛЕНАМИ МЕНЬШЕГО ПОРЯДКА

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РЕЗЮМЕ

В статье рассматриваются недивергентные вырождающиеся эллиптические уравнения второго порядка. Получены достаточные условия для устранимости компакта относительного этих уравнений в весовых пространствах Гельдера.

Ключевые слова: устранимые особенности, решение, вырождающийся, уравнение.

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