INFRARED RENORMALONS AND SINGLE MESON PRODUCTION IN PROTON-PROTON COLLISIONS

A. I. Ahmadov, Sh. M. Nagiyev*, E. A. Dadashov*
Baku State University
Z.Khalilov st. 23, Az-1148 Baku, Azerbaijan
**Institute of Physics, ANAS*

The large-order behavior of a perturbative expansion in gauge theories is inevitably dominated by the factorial growth of renormalon diagrams [1-4]. In the case of quantum chromodynamics (QCD), the coefficients of perturbative expansions in the QCD coupling $\alpha_s(-k^2)$ can increase dramatically even at low orders. This fact, together with the apparent freedom in the choice of renormalization scheme and renormalization scales, limits the predictive power of perturbative calculations, even in applications involving large momentum transfer, where α_s is effectively small. It is well known, that in pQCD calculations the argument of the QCD coupling constant (or the renormalization and factorization scale) $\hat{Q}^2$ should be taken equal to the square of the momentum transfer of a hard gluon in a corresponding Feynman diagram. But defined in this way, $\alpha_s(\hat{Q}^2)$ suffers from infrared singularities in the end –point regions and for removing appeared divergences the QCD running coupling (RC) method [5] and technique of IR calculus [4] should be applied. Such infrared singularities appear also in calculation of other processes. For example in our work [6], $\hat{Q}^2$ equals to $(x_1-1)\hat{u}$ and $x_1\hat{t}$, where $\hat{u}$, $\hat{t}$ are the subprocess's Mandelstam invariants. In our calculations the factorization scale does not depend on x, $Q^2=p_T^2$. Such approximation does not change considerably numerical results, but phenomenon considering in this article (effect of infrared renormalons) becomes transparent. Therefore, in the soft regions $x_1\to 0$, $x_2\to 0$ integrals (2.15) in [6] diverge and for their calculation regularization methods of Ref [5] are needed.

Investigation of the infrared renormalon effects in various inclusive and exclusive processes is one of the important and interesting problems in the perturbative QCD. It is known, that infrared renormalons are responsible for factorial growth of coefficients in perturbative series for the physical quantities. But, these divergent series can be resummed by means of the Borel transformation [1] and the principal value prescription and running coupling effects can be taken into account by scale-setting procedure $\alpha_s(Q^2)\to\alpha_s(\exp(f(Q^2))Q^2)$ at the one-loop order results. Technically, all-order resummation of infrared renormalons corresponds to the calculation of the one-loop Feynman diagrams with the running coupling $\alpha_s(-k^2)$ at the vertices or, alternatively, to calculation of the same diagrams with non-zero gluon mass. Studies of infrared renormalon problems have opened also new prospects for evaluation of power suppressed corrections to processes’ characteristics.

By taking these points into account, it may be asserted that the analysis of the higher twist effects on the dependence of the pion wave function in pion production at proton-proton collisions by the RC method, where these singularities had been regularized by means of the principal value prescription are significant in both theoretical and experimental studies. Another important aspect of this study is the choice of the meson model wave functions. The hadron wave function gives the amplitude for finding partons (quarks, qluons) carrying the longitudinal fractional momenta $\mathrm{x}=(x_1,x_2,...x_n)$ and virtualness up to Q^2 within the hadron and, in general, includes all Fock states with quantum numbers of the hadron. But only the lowest Fock state ($q_1\bar{q}_2$-for mesons, *uud* -for proton, *etc.*) contributes to the leading scaling behavior, other Fock state contributions are suppressed by powers of $1/Q^2$. In our work, we have restricted ourselves to considering the lowest Fock state for a meson. Then $x = x_1, x_2$ and $x_1 + x_2 = 1$.

In this work, we investigate the contribution of the high twist Feynman diagrams to the large- p_T inclusive pion production cross section in proton-proton collisions and we present the general formulae for the high twist differential cross sections in case running coupling and frozen coupling approaches.

The cross section for the high twist subprocess is given by the expression

$$\frac{d\sigma}{d\hat{t}} = \frac{8\pi^2 \alpha C_F}{27} \left[D(t,\hat{u})\right]^2 \frac{1}{\hat{s}^3(-\hat{t})} \left[\frac{1}{t^2} + \frac{1}{\hat{u}^2}\right], \tag{1}$$

where

$$D(\hat{t},\hat{u}) = e_1 \hat{t} \int_0^1 dx_1 \left[\frac{\alpha_s(Q_1^2)\Phi_\pi(x_1,Q_1^2)}{1-x_1}\right] + e_2 \hat{u} \int_0^1 dx_1 \left[\frac{\alpha_s(Q_2^2)\Phi_\pi(x_1,Q_2^2)}{1-x_1}\right], \tag{2}$$

and $Q_1^2 = (x_1 - 1)\hat{u}$, $Q_2^2 = -x_1 \hat{t}$ represent the momentum squared carried by the hard gluon, $e_1(e_2)$ is the charge of $q_1(\bar{q}_2)$ and $C_F = \frac{4}{3}$.

The main problem in our investigation is the calculation of (1) by RC method. The integral in Eq.(1) in the framework of the running coupling method takes the form

$$I(Q^2) = \int_0^1 dx_1 \left[\frac{\alpha_s(\lambda Q^2)\Phi_M(Q^2,x)dx}{1-x}\right]. \tag{3}$$

The $\alpha_s(\lambda Q^2)$ has the infrared singularity at $x \to 1$, if $\lambda = 1 - x$ and as a result integral (3) diverges (the pole associated with the denominator of the integrand is fictitious, because $\Phi_M \sim (1-x)$, and therefore, the singularity of the integrand at $x = 1$ is caused only by $\alpha_s((1-x)Q^2)$. For the regularization of the integral we relate the running coupling at scaling variable $\alpha_s(\lambda Q^2)$ with the aid of the renormalization group equation in terms of the fixed one $\alpha_s(Q^2)$.

The structure of infrared renormalon singularities of the higher twist subprocess cross section and the resummed expression (the Borel sum) for it are found. We compared the resummed high twist cross sections with the ones obtained in the framework of the frozen coupling approximation and leading twist cross section. We obtain, that ratio R at all values of the transverse momentum p_T of the pion is equivalent to ratio r. It is shown that the resummed result depends on the choice of the meson wave functions used in calculation. Phenomenological effects of the obtained results have been discussed.

REFERENCES

[1] G. t'Hooft, in The Whys of Subnuclear Physics, Erice, 1977, edited by A. Zichichi (Plenum, New York, 1979), 94.

[2] A. H. Muller, Nucl. Phys. **B250**, 327 (1985); Phys. Lett. **B308**, 355 (1993).

[3] V. I. Zakharov, Nucl. Phys. **B385**, 452 (1992).

[4] M. Beneke, Phys. Rep. **317**, 1 (1999).

[5] S. S. Agaev, Phys. Lett. **B360,** 117 (1995); ICTP preprint IC/95/291, hep-ph/9611215.

[6] A. I. Ahmadov, I. Boztosun, R. Kh. Muradov, A. Soylu and E. A. Dadashov, Int. J. Mod. Phys. **E15**, 1209 (2006).

[7] H. Contopanagos and G. Sterman, Nucl. Phys. **B419**, 77 (1994).