Multi-particle equations in quantum field theory models and their possible applications

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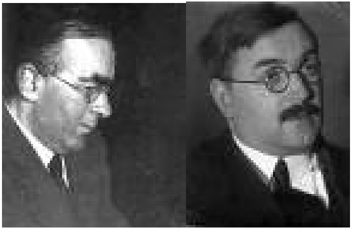
Introduction and Summary

The present report is designed to focus attention on the Hartree-Fock



approximation

$$\hat{F}\varphi_i = \varepsilon_i \varphi_i,$$

as a general basic for construction some non-perturbative approaches for describe of dynamics of strong interaction systems.

Introduction and Summary

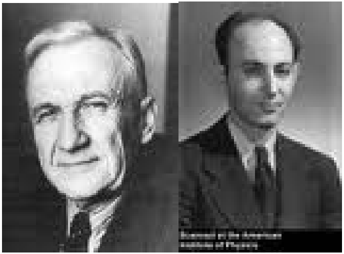
Ever since of success of the Tomonaga-Schwinger-Feynman-Dyson

formalism in QED, corresponding field-theoretic formulations have been in the forefront of strong interaction dynamics since the early fifties of past century, the main strategy being to device various 'closed' form of approaches which are represented as appropriate 'integral' equations.

Introduction and Summary

One of the earliest efforts in this direction was the Tamm-Dancoff formalism,

I.E. Tamm:*J. Phys. 9:449, 1945;*
S.M. Dancoff:Phys. Rev. 78:382, 1950



which showed a great intuitive appeal.

Introduction and Summary

In this method, the state vector of the system under consideration is Fock-expanded in terms of a complete set of eigen-functions of the free field hamiltonian, which was first systematically applied by Dyson (+ Cornell collaborators) in the early fifties, to the meson-nucleon scattering problem, for a dynamical understanding of the 'Delta' and other low-energy resonances

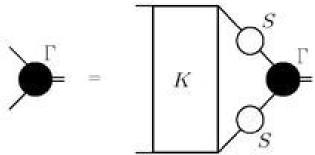
H.A. Bethe and F. de Hoffmann: *Mesons and Fields II, Row, Peterson and Co, N.Y.,1955, p.199;*
V.P. Silin, I.Y. Tamm, V.Ya. Fainberg: ZhETF 29:6, 1955.


Introduction and Summary

The 3D Tamm-Dankoff equation and the 4D Schwinger-Dyson equation (SDE) have been the source of much wisdom underlying the formulation of many approaches to strong interaction dynamics. To these one should add the Bethe-Salpeter equation (BSE),

E.E. Salpeter and H.A. Bethe: *Phys. Rev. 84:1232, 1951*

Introduction and Summary



which is an approximation to SDE for the dynamics of a 4D two-particle amplitude, characterized by an effective (gluon-exchange-like) pairwise interaction, for the effective N-N interaction, but now adapted to the quark level.

A major bottleneck for the BSE approach has been its resistance to a probability interpretation, since the logical demands of its 4D content are incompatible with its approximate nature. This has led to Instantaneous approximation

M. Levy: *Phys. Rev. 88:72, 1952;*

variants of on-shellness of the associated propagators

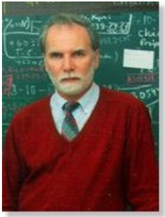
R. Blakenbecler and R. Sugar: Phys. Rev. 142:105, 1966.

and to Logunov-Tavkhelidze Quasi-potential equation

A.A. Logunov and A.N. Tavkhelidze: *Nuovo Cimento 29:380, 1963*

and Kadyshevsky formalism:

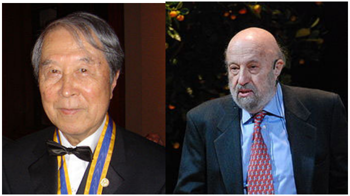
V.G. Kadyshevsky: *Sov Phys. JETF 19:443;597, 1964,;* Nucl. Phys. B6:125, 1968,;
C. Itzikson, V.G. Kadyshevsky,I.T. Todorov: *Phys. Rev. D1:2823, 1970.*



Introduction and Summary

Above-named equations have been widely employed as prototypes of strong interaction dynamics, addressing issues of gauge and chiral symmetries, as well as dynamical breaking of chiral symmetry (DBCS) via an NJL-type mechanism

Y. Nambu and G. Jona-Lasinio: *Phys. Rev. 122:345, 1961; 124:246,1961.*



Introduction and Summary

The report is organized as follows:
In sections 2-3 we results the solutions of ladder perturbative BSE. And discuss about multifermion equations in QED.
In Section 4 we describe the method of construction of the MFE with the fermion bilocal source for the NJL model with the $SU_V(2) \times SU_A(2)$–symmetric four-quark interaction and, for the sake of completeness, consider the well-known leading approximation results of this model. Also in this section we investigate the first-after-leading step of the iteration scheme, which gives us the equations for the leading order two-particle Green function and NLO correction to the propagator of quarks.
In Section 5 we describe the second step of the iteration scheme. As a result we obtain the equations for four-quark Green function and for the three-quark Green function. We also obtain in this step the equations for NLO two-quark function and NNLO correction to quark propagator. We discuss the structure of second step equations and obtain the solutions of four-quark and three-quark equations.

In Section 6 we describe the third step of iteration scheme. As a result we obtain the equations for six-quark Green function and for the five-quark Green function, and, the NLO equations for four-quark and three-quark Green functions. We also obtain in this step the equations for NNLO two-particle function and NNNLO correction to quark propagator.
In Section 7 the modification of the MFE for the NJL model in the formalism with the multilocal diquark and triple-quark sources is briefly discussed.

Section 2. The ladder Bethe-Salpeter equations and their possible solutions

The ladder approximation in method of BSE for the scattering amplitude in field theory models was originally used justify Regge behavior at high energies

B.A. Arbuzov, A.A. Logunov, A.N. Tavkhelidze, R.N. Faustov: *Phys. Lett. 2:150, 1962;*
J.C. Polkinghorne: J. Math. Phys.4:503, 1963;
D. Amati, S. Fubini, A. Stanghellini: *Nuovo Cim.26:896, 1962;*
L. Bertocchi, S. Fubini, M. Tonin: Nuovo Cim.25:626, 1962

and was the point of departure in the construction of the multi-peripheral model.

Section 2.

Different methods have been used to obtain exact solutions of ladder BSE for forward scattering amplitude in a number of models and other works by B.A. Arbuzov and Co

B.A. Arbuzov, V.E. Rochev: *Yad.Fiz. 21:883, 1975;*
B.A. Arbuzov, V.Yu. Diakonov, V.E. Rochev: Yad.Fiz. 23:904, 1976;
K.G. Klimenko, V.E. Rochev: *Yad.Fiz. 31:448, 1980;*
V.Yu. Diakonov: TMF 43:218, 1980

and

C.G. Callan, M.L. Goldberger: *Phys. Rev. D11:1553, 1975*
I.J. Muznich, H.S. Tsao: Phys. Rev. D11:2203, 1975

and the behavior of these solutions in both the Regge and deep inelastic (Bjorken) regions has been investigated.

Section 2.

In particular for ladder BSE for imaginary part of scattering amplitude
$p + p' = k + k'$

$$F(s,t) = \pi\lambda^2\delta(s-\mu^2) + \frac{\pi\lambda^2}{(2\pi)^4}\int d^4q \frac{\theta(s'-q_0)\delta(q^2-\mu^2)}{[(p-q)^2-m^2][(k-q)^2-m^2]} F(s',t) \quad (1)$$

Here $s = (p+p')^2$, $s' = (p+p'-q)^2$, $t = (p-k)^2$ and μ is exchange mass, and m_0 is the mass in other propagators,
$d^4q = dq_0|\overrightarrow{q}|d|\overrightarrow{q}|dcos\theta d\varphi$.

Section 2. Forward scattering:

In these works to find via different mathematical way (the inverse Mellin transformation,a and/or via diagonalized way by means of an expansion in Gegenbauer polynomials) to find the solutions in high energies $s \to \infty$, and at $t = 0$ - only forward scattering, in common approximately in form

$$F(s') = C(g^2, \alpha)(\frac{s}{m^2})^{\alpha},$$

where $g^2 = \frac{\lambda^2}{32\pi^2 m^2}$, and Regge parameter α has the form

$$\alpha = -\frac{1}{2} + \sqrt{\frac{1}{4} + g^2}.$$

Such result lead us to idea, which consist in finding the solution in starting as Regge form of behavior of scattering amplitude.

Ceremony of award to Regge and L.N. Lipatov Pomeranchuck price

Subsection 2.1. Easy way for solution of ladder BSE for imaginary part of forward scattering amplitude

Let us to introduce in kernel of integral (1) a one as integral $1 = \int \delta((p+p'-q)^2 - s')ds'$. In case of forward scattering $p = k$, $p' = k'$ and $p^2 = m^2$, the integration with respect to φ, $dcos\theta$, dq_0 and $d|\overrightarrow{q}|$ in c.m.s. $|\overrightarrow{p}| + |\overrightarrow{p'}| = 0$ is trivial. The result is

$$F(s) = \frac{\pi^2\lambda^2}{2(2\pi)^4 m^2} \int d(\frac{s'}{s}) \frac{(1-\frac{s'}{s})F(s')}{(1-\frac{s'}{s})^2 + \frac{\mu^2}{m^2}}$$

Let us to find the solution as

$F(s) = s^{\alpha}$.

The result of integration is the sum of two hypergeometric equations

$$64\pi^2\mu^2 \frac{(\alpha+1)(\alpha+2)}{\lambda^2} = F(1,2;\alpha+3;-i\frac{m}{\mu}) + F(1,2;\alpha+3;i\frac{m}{\mu}). \quad (2)$$

Subsection 2.1. Forward scattering

1) In case $m << \mu$,

$$\alpha = -\frac{3}{2} \pm \frac{1}{2}\sqrt{1 + \frac{\lambda^2}{8\pi^2\mu^2}}.$$

2)At $\mu << m$

$$\alpha = -n \pm (-\frac{32\pi^2 m^2}{\lambda^2} + \frac{1}{2} ln \frac{\mu^2}{m^2}), \quad n = 1, 2, 3, ...$$

S.A. Gadjiev, R.G. Jafarov: *Dokl.AN Azerb. , v.XLII: No11: 20, 1986;*
S.A. Gadjiev, R.G. Jafarov: Dokl.AN Azerb. , v.XLIII: No1:34, 1987.

Subsection 2.2. Small momentum transfers

The Eq.(1) after the integration with respect to φ, $dcos\theta$, dq_0 and $d|\overrightarrow{q}|$ in c.m.s. $|\overrightarrow{p}| + |\overrightarrow{p'}| = 0$, in case $p = k$, $p' = k'$ and $p^2 = m^2$ and $k^2 = m^2$ receive the form

$$F(s,t) = \frac{\pi^2\lambda^2}{8(2\pi)^4|\overrightarrow{p}|^2\sqrt{s}\sqrt{\frac{(s-s'+\mu^2)^2}{4s} - \mu^2}}\cdot$$

$$\cdot\int ds'dz\frac{F(s',t)}{(\beta+z)(z^2+2\beta z_0 z+\beta^2+{z_0}^2-1)^{1/2}},$$

where $z = cos\theta$, $z_0 = cos\theta_0$, θ_0-the scattering angle and $\beta = \frac{\mu^2-s+s'}{4|\overrightarrow{p}|\sqrt{\frac{(s-s'+\mu^2)^2}{4s}-\mu^2}}$.
Let us take in place $z_0 = 1+\epsilon$, where $\epsilon << 1$ and to expand to series. We find the solution as $F(s,t) = s^{\alpha,t}$. The result of integrations is

$$64\pi^2\mu^2\frac{(\alpha+1)(\alpha+2)}{\lambda^2(1+\frac{t}{6m^2})} = F(1,2;\alpha+3;-i\frac{m}{\mu}) + F(1,2;\alpha+3;i\frac{m}{\mu}). \quad (3)$$

Subsection 2.2.

1) In case $m << \mu$,

$$\alpha = -\frac{3}{2} \pm \frac{1}{2}\sqrt{1 + \frac{\lambda^2}{8\pi^2\mu^2}(1 + \frac{t}{6m^2})}.$$

2)At $\mu << m$

$$\alpha = -n \pm (-\frac{32\pi^2 m^2}{\lambda^2(1 + \frac{t}{6m^2})} + \frac{1}{2} ln \frac{\mu^2}{m^2}), \;\; n = 1, 2, 3, ...$$

In case $t = 0$ all results have exact co-ordinate with results of forward scattering.

S.A. Gadjiev, R.G. Jafarov: *Krat. Soobsh. po Fizike FIAN, No11:25, 1986.*
S.A. Gadjiev, R.G. Jafarov, A.I. Livashvili: Izvest. Vuzov. Fizika No5:49, 1989.

Section 3. Multi-particle equations

The multi-particle (three or more particle) generalizations of the 4D BSE have been studied in detail. A straightforward generalization of two-particle BSE has bee intensively studied in sixties-seventies of last century. A best exposition of these studied can be found in the work of Huang and Weldon

K. Huang and H.A. Weldon: *Phys. Rev.D11:257, 1975.*

These generalizations are based on the analysis of Feynman diagrams, and all statements have a perturbative sense only. A form of the equations was chosen arbitrary. (This arbitrariness is connected with a choice of an inhomogeneous term). In addition, one can prove any proposition by the diagrammatic method in a framework of some model only, and the question about the model-independence is inevitable. An additional disadvantage of the diagrammatic method is the fact almost all propositions can be formulated in words and cannot be formalized.

Section 3.

The above-mentioned difficulties cannot be resolved in the framework of the diagrammar. However, the natural language exists for the description multi-particle equations in the framework of the Lagrangian field theory. There are Legendre transformations of the generating functional for the Green's functions.
Functional Legendre transformations were firstly introduced in quantum statistics

C. De Dominicis: *J. Math. Phys.3:983, 1962;*
C. De Dominicis and P.C. Martin: J. Math. Phys.5: 31 1964.

Then they were applied to the quantum field theory

Dahmen H.D., Jona-Lasinio G: *Nuovo Cim. A52:807, 1967;*
Vassiliev A.N.: Functional methods in quantum field theory and statistics.*Leningrad, 1976;*
*Rochev V.E.:*Teor. Mat. Fiz. 51:22, 1982.

Section 3.

With these transformations one can obtain multi-particle equations as a consequence of Schwinger ones. These multi-particle equations are model-independent, and they do not depend on perturbation theory. Any dynamical information about an interaction is contained in the equation kernel.
A number of perspective physical applications of the effective models are connected with multi-particle functions, which are, in the main, the subject of present report. The basic method of calculations is a formalism of multilocal sources.

Section 3.1. New non-perturbative method in QED and the multi-fermion equations

The problem of nonperturbative calculations in QED arose practically simultaneously with the principal solution of the problem of perturbative calculations with based on renormalized coupling constant perturbation theory. It is necessary to recognize, however, that the progress in the nonperturbative calculations during last decades is not to large. Quantative description of nonperturbative effects either is based on non-relativistic foundations (an example is the bound state description based on non-relativistic Coulomb problem) of is rather open to injury for a criticism. Besides, the problem of inner inconsistency of QED exist. This problem can be formulated as a deep-rooted thesis on triviality of QED in the nonperturbative region. A new approach to nonperturbative calculations in quantum electrodynamics is proposed in work

Rochev V.E.:*J.Phys. A33:7379, 2000.*

Section 3.1.

This approach is based on a regular iteration scheme for the solution of Schwinger-Dyson equations for generating the functional of Green functions of QED by an exactly soluble equation. Its solution generates a linear iteration scheme each step of which is described by a closed system of integro-differential equation.

Note that equations of Green function at leading approximation and at the first step of iteration scheme in two versions. First of them on the language of Feynman diagrams of perturbative theory is analog of summation of chain diagrams with fermion loop.

The second version of the iteration scheme can be compared on the diagram language a ladder summation. The generating functional has the form

$$G(J,\eta) = \int D(\psi,\bar{\psi},A) expi\{\int (L + J_\mu(x)A_\mu(x)) - \int dxdy\bar{\psi}^\beta\eta^{\beta\alpha}(y,x)\psi^\alpha(x)\}.$$

Functional derivatives of G with respect to sources are vacuum expectation values. SDEs for the generating functional of Green functions of QED has the forms:

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{d_l}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G}{\delta J_\nu} + ietr\left[\gamma_\mu \frac{\delta G}{\delta \eta}\right] + J_\mu G = 0, \quad (4)$$

$$G + (i\widehat{\partial} - m)\frac{\delta G}{\delta \eta} + \frac{e}{i}\gamma_\mu \frac{\delta^2 G}{\delta J_\mu \eta} - \eta \star \frac{\delta G}{\delta \eta} = 0. \quad (5)$$

In correspondence with the choice of the leading approximation i-th term of the iteration expansion of the generating functional

$$G = G^{(0)} + G^{(1)} + G^{(2)} + \cdots, \quad (6)$$

which is solution of iteration scheme equations. A solution of equations (4), (5) is looked in the form:

$$G^{(i)} = P^{(i)} G^{(0)}.$$

Subsection 3.1.1. Chain approximation

Since $P^{(0)} \equiv 1$, it is evident that for any "i" the functional $P^{(i)}$ is a polynomial in functional variables J and η. This circumstance is very important since it means the system of equations for coefficient functions of this functional take closed in any order of the iteration scheme.
This iteration scheme has no explicit small parameter. In some sense, the sources J and η play the role of such a parameter. Expansion (6) of the generating functional should be treated as an approximation of $G(J,\eta)$ near the point $J_\mu = 0$, $\eta = 0$.
The iteration equation for the generating functional of Green functions of chain approximation in switching off photon sources $J_\mu = 0$ has the form:

$$G^{(i)} + (i\widehat{\partial} - m_0)\frac{\delta G^{(i)}}{\delta\eta} - ie^2\{D_{\mu\nu}\star\gamma_\mu\frac{\delta}{\delta\eta}tr[\gamma_\nu\frac{\delta G^{(i)}}{\delta\eta}]\} = \eta\star\frac{\delta G^{(i-1)}}{\delta\eta}. \quad (7)$$

The solution of first step equation is

$$G^{(1)} = \left\{\frac{1}{2}S_2\star\eta^2 + S^{(1)}\right\}$$

Subsection 3.1.1.

Let us use the following Feynman graphical rules

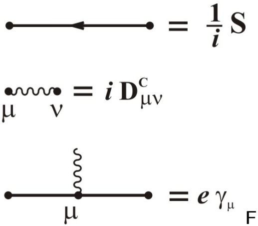


Figure 1.

we obtain the BSE in chain approximation

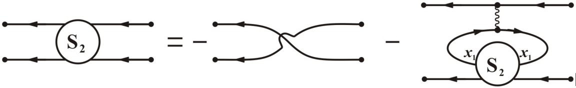


Figure 2.

as means as series (see Fig.2)

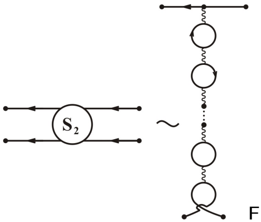


Figure 3.

The second step generation functional is

$$G^{(2)}(\eta) = P^{(2)}(\eta)G^{(0)}, \tag{8}$$

where

$$P^{(2)} = \frac{1}{4!}S_4 \star \eta^4 + \frac{1}{3!}S_3 \star \eta^3 + \frac{1}{2}S_2^{(1)} \star \eta^2 + S^{(2)} \star \eta.$$

Subsection 3.1.1.

The second iteration step contains the equations for the four S_4- and three S_3 functions and also the equations for the first order correction to two-fermion function $S_2^{(1)}$ and second-order correction equation to electron propagator $S^{(2)}$. For these four functions we have a system of four integral equations, which , and all equations, (also for next, ladder approximation equations) posses the similar structure.

$$S_n = S_n^0 - ie^2 \left\{ (D_{\mu\nu}^c \star S \cdot \gamma_\mu S) \star tr[\gamma_\nu S_n] \right\}$$

and differ from each other by the structure of inhomogeneous terms S_n^0.

Subsection 3.1.1.

The inhomogeneous term S_4^0 for four-electron function is

$$S_4^0 = -3 \cdot \{S \cdot S \cdot S_2\},$$

where S_2 is very well known form.

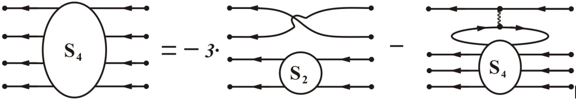


Figure 4.

The inhomogeneous term of three-electron equation is

$$S_3^0 = -2 \cdot \{S \cdot S \cdot S^{(1)}\} - 2 \cdot \{S \cdot S_2\} - ie^2 \left\{(D^c_{\mu\nu} \star S\gamma_\mu) \star tr[\gamma_\nu S_4]\right\}.$$

Here $S^{(1)}$ is first step correction electron function, which is defined in preceding step.

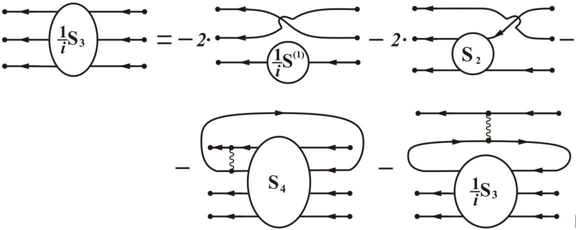


Figure 5.

Subsection 3.1.1.

The inhomogeneous term of the first order correction for two-electron function has the following form

$$(S_2^{(1)})^0 = -\{S \cdot S^{(1)}\} - ie^2 \left\{(D^c_{\mu\nu} \star S\gamma_\mu) \star tr[\gamma_\nu S_3]\right\},$$

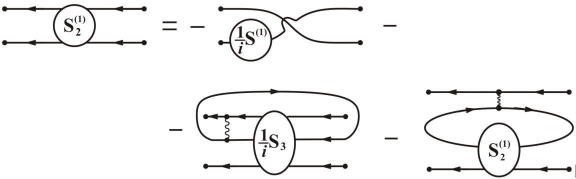


Figure 6.

and the inhomogeneous term of second-order correction for single electron function absence

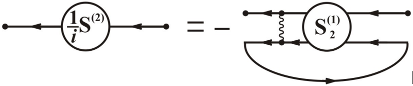


Figure 7.

Subsection 3.1.2. Ladder BSE

As we note the leading order and first step equations are very well known

Rochev V.E.:*J.Phys. A33:7379, 2000.*

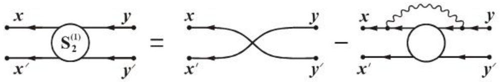
Here we would like to demonstrate the solution of ladder BSE for two-electron bound state and the constructing of second order equations.

Jafarov R.G.:*Izv. Akad. nauk Azerb. 25, No5:19, 2005;*
Gadjiev S.A., Jafarov R.G.:Izv. Akad. nauk Azerb. 26, No5:20, 2006.

This step leads us very to well known two-electron function equation is

$$S_2 = -S \cdot S + K \star S_2$$

where $K = ie^2\{tr[D_{\mu\nu} \star SS\gamma_\mu S_2 \gamma_\nu S],\}$ is the kernel of equation.

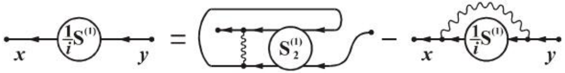


Subsection 3.1.2. Ladder BSE

The equation for first step electron propagator is

$$S^{(1)} = ie^2 D^c_{\mu\nu} \star S\gamma_\mu S_2 \gamma_\nu + ie^2 D^c_{\mu\nu} \star S\gamma_\mu S^{(1)} \gamma_\nu S$$

which have a following graphical form



Subsection 3.1.2. Ladder BSE

BSE in momentum space is

$$S^{-1} \cdot S_2 \cdot S^{-1} = 1 \cdot 1 + ie^2 D^c_{\mu\nu} \star \gamma_\mu S_2 \gamma_\nu$$

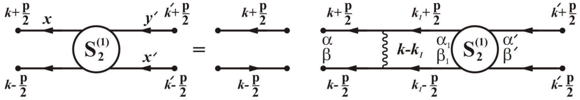
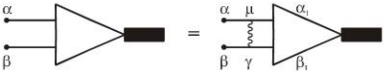


Figure 10.

The BSE for bound states is

$$S^{-1} \chi^{\alpha\beta} S^{-1} = ie^2 D^c_{\mu\nu} \star \gamma_\mu \chi^{\alpha\beta} \gamma_\nu$$



Section 4. Mean-field expansion for Nambu–Jona-Lasinio model and the multi-quark functions

A number of perspective physical applications of the effective models are connected with multi-quark functions, which are the subject of present report. The basic method of calculations is a formalism of multilocal (double, triple, etc.) sources

Rochev V.E.: *Teor. Mat. Fiz. 51:22, 1982.*

As an object of application of the method we choose Nambu - Jona-Lasinio (NJL) model
This model is one of the most successful effective models of quantum chromodynamics for the light hadrons. For review see

Klevansky S.P.: *Rev. Mod. Phys. 64:649, 1992;*
Hatsuda T. and Kunihiro T.: Phys. Reports 247:221, 1994;
Volkov M.K., Radjabov A.E.: *Uspekhi Fiz. Nauk 176:569, 2006.*

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Section 4.

It is necessary to note, that this method has been successfully applied for the other field-theoretic models and can be applied also for analogous calculations in other similar effective models.
The multi-quark functions arise in higher orders of the mean-field expansions (MFE) for the NJL model. To formulate the MFE we have used an iteration scheme of solution of the Schwinger-Dyson equation with the fermion bilocal source, which has been developed in works by Rochev. We have considered the equations for Green functions of the NJL model up to the third order of the MFE. The leading approximation and first order of the MFE maintains equations for the quark propagator and the two-quark function and also the NLO correction to the quark propagator. The second order of MFE includes the equations for the four-quark and the three-quark functions and also the equations for the NLO two-quark function and NNLO quark propagator.

Section 4.

Furthermore we have considered the generalization of the method in the framework of the NJL-type models, which includes the other multilocal sources (specifically, the diquark and three-quark sources).
We have found a solution of the four-quark and three-quark equations. The solution of the four-quark function is a disconnected combination of the leading-order functions and, consequently, the corresponding physical effects (i.e., pion-pion scattering) are suppressed in this order of the MFE. Therefore, we also investigate the third step of iterations, which gives us the equations for the six-quark and five-quark functions and the equations for the NLO four-quark and three-quark functions. The solution of the six-quark functions equation has the disconnected form, which is similar to the solution for the four-quark function of the preceding step. The solution of the second-step four-quark equation gives us a possibility to close the equation for the three-quark function.

Subsection 4.1. The method. Leading order and first step equations

The Lagrangian of the two-flavor NJL model may be written in the well-known form

$$L = \bar{\psi} i \hat{\partial} \psi + \frac{g}{2}\Big[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau^a \psi)^2 \Big]. \tag{9}$$

To construct the MFE we use an iteration scheme of the solution of functional-differential SDE

$$G + i\hat{\partial}\frac{\delta G}{\delta \eta} + ig\Big\{ \frac{\delta}{\delta \eta} tr\Big[\frac{\delta G}{\delta \eta}\Big] + i\gamma_5 \tau^a \frac{\delta}{\delta \eta} tr\Big[i\gamma_5 \tau^a \frac{\delta G}{\delta \eta}\Big]\Big\} = \eta \star \frac{\delta G}{\delta \eta} \tag{10}$$

for the generating functional G of Green functions.

Subsection 4.1.

The generating functional G can be represented as the functional integral with bilocal fermion source η:

$$G(\eta) = \int D(\psi, \bar{\psi}) \exp i\Big\{ \int dx L - \int dx dy \bar{\psi}(y)\eta(y,x)\psi(x)\Big\}. \quad (11)$$

We shall solve Eq. (10) employing the method which proposed in work by

Rochev V.E., Jafarov R.G.:*Central Eur. J. Phys. 2:367, 2004 (arXiv:hep-ph/0311339).*

The solution of the equation of leading approximation,i.e., the functional-differential SDE (10) with zero r.h.s., is the following functional $G^{(0)} = \exp\left\{\mathrm{Tr}\left(S \star \eta\right)\right\}$, where S is solution of the equation

$$1 + i\hat{\partial}S + igS \cdot tr[S(0)] = 0. \quad (12)$$

The leading approximation generates the linear iteration scheme

$$G = G^{(0)} + G^{(1)} + \cdots + G^{(n)} + \cdots ,$$

consists in the step-by-step solutions of the equations

$$G^{(n)} + i\hat{\partial}\frac{\delta G^{(n)}}{\delta\eta} + ig\left\{\frac{\delta}{\delta\eta}tr\left[\frac{\delta G^{(n)}}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\eta}tr\left[\gamma_5\tau^{\mathbf{a}}\frac{\delta G^{(n)}}{\delta\eta}\right]\right\} = \eta\star\frac{\delta G^{(n-1)}}{\delta\eta}. \tag{13}$$

Functional $G^{(n)}$ is $G^{(n)} = P^{(n)}G^{(0)}$, where $P^{(n)}$ is a polynomial of $2n$ -th degree on the bilocal source η.
The unique connected Green function of the leading approximation S is the quark propagator. A solution of Eq. (12) is

$$S(p) = (m - \hat{p})^{-1},$$

where m is the dynamical quark mass, which is a solution of the gap equation of the NJL model in the chiral limit.

Subsection 4.1.

The other connected Green's functions appear in the subsequent steps of the iterative scheme.
The first iteration step contains the leading-order equation for the two-particle S_2 quark function

$$S_2 = -S \cdot S + K \star S_2 \tag{14}$$

$K = ig\Big\{(S \cdot S) \star tr[S_2] - (S\gamma_5\tau^a S) \star tr[\gamma_5\tau^a S_2]\Big\}$ is the kernel of equation

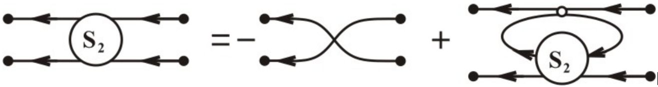


Figure 12.

and first order quark function equation

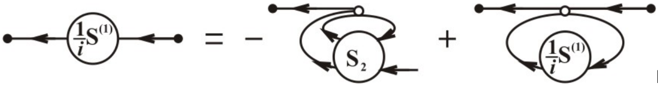


Figure 13.

Subsection 4.2. Dimensional-analytically regularized NJL model and NJL model with 4- cutoff regularization. Gap equations

Since the NJL model in the mean-field approximation includes quark loops, the essential aspect of application of this model is a regularization. Most common regularizations for NJL model traditionally entail a 4D cutoff in Euclidean momenta or 3D momentum cutoff. Other regularization schemes (Pauli-Villars regularization or non-local Gauss formfactors) also are used for the NJL model. The least common regularization for NJL model is a dimensional regularization. Thus in reviews cited above,

Klevansky S.P.:*Rev. Mod. Phys. 64:649, 1992;*
*Hatsuda T. and Kunihiro T.:*Phys. Reports 247:221, 1994

this regularization is not even mentioned.
The parameter of dimensional regularization, which traditionally treated as a deviation in physical dimension of space,does not permit any physical interpretation in this treatment.

Subsection 4.2.

However, an alternative treatment of dimensional regularization exists - as a variant of an analytical regularization. In this treatment all calculations are made in 4-dimensional Euclidean momentum space, and the regularization parameter is treated as a power of a weight function

$$w_{\Lambda,D(q_e^2)} = w_\Lambda(q_e^2) w_D(q_e^2) = \theta(\Lambda^2 - q_e^2) \left(\frac{\mu^2}{q_e^2} \right)^{2-D/2},$$

$$(\mu^2)^{2-D/2} = \frac{\Omega_D}{\Omega_4} \frac{(2\pi)^4}{(2\pi)^D} (M^2)^{2-D/2}, \tag{15}$$

which regularizes divergent integrals.

Krewald S. and Nakayama K. *Annals of Phys.216:201, 1992.*

It should be stressed that in this treatment of dimensional regularization, the regularization parameter is not at all a deviation in the physical dimension of space. Can be to suppose that a possible treatment of this parameter is a power of some factor, which is a measure of gluon influence on the effective four-fermion quark self-action of NJL model, which has something in common with non-local variants of the NJL model.
For dimensionally-analytical regularization the gap equation has the form

$$1 = \kappa\Gamma(\xi) \tag{16}$$

where $\xi = 1 + \frac{D}{2}, D = 2 - 2\xi$ and $\kappa = \frac{g n_c m^2}{2\pi^2}$.
And in for 4-dimensional cutoff regularization

$$1 = \kappa_\Lambda \left(1 - \frac{m^2}{\Lambda^2} \log\left(1 + \frac{\Lambda^2}{m^2}\right)\right), \tag{17}$$

where $\kappa_\Lambda = \frac{g n_c \Lambda^2}{2\pi^2}$.

Subsection 4.3. Two-particle amplitude and model parameters in leading approximation

Scalar and pseudoscalar amplitudes

$$A_\sigma = \frac{1}{4n_c\left(4m^2 - p^2\right) I_0}, \tag{18}$$

$$A_\pi = \frac{1}{4n_c p^2 I_0}, \tag{19}$$

where

$$I_0\left(p^2\right) = \int \frac{d\,\widetilde{q}}{\left(m^2 - (p+q)^2\right)\left(m^2 - q^2\right)}, \tag{20}$$

which are in both regularization as following:

$$[I_0\left(p^2\right)]^{DAR} = \frac{i}{(4\pi)^2}\frac{\xi}{\kappa}F\left(1+\xi, 1; 3/2; \frac{p^2}{4m^2}\right), \tag{21}$$

$$\begin{aligned}[I_0\left(p^2\right)]^{FDC} = \frac{i}{(4\pi)^2}\Bigg[\log\left(1+x\right) - \\ -\frac{x}{1+x}F\left(1,1;3/2;\frac{p^2}{4m^2\left(1+x\right)}\right) - \\ -\frac{p^2}{6m^2(1+x)}F\left(1,1;5/2;\frac{p^2}{4m^2\left(1+x\right)}\right) + \\ +\frac{p^2}{6m^2}F\left(1,1;5/2;\frac{p^2}{4m^2}\right)\Bigg].\end{aligned} \tag{22}$$

Here $F(a, b; c; z)$ is the Gauss hypergeometric function.

As a measure of quantum fluctuations of the chiral field, consider a ratio of first-order condensate to the leading-order condensate

$$r \equiv \frac{\chi^{(1)}}{\chi^{(0)}} = \frac{i\mathrm{tr}S^{(1)}(0)}{i\mathrm{tr}S_0(0)} = r_\sigma + r_\pi. \qquad (23)$$

The results of parameter fixing in the leading approximation are given in Table.1 and Table.2. As it is seen from these Tables the value of the main parameter - quark mass m in model with 4D cutoff is much more sensitive to the value of chiral condensate in comparison with that of the model with DAR. At the same time it is necessary to point, that there are no some principal distinctions of this variants of the NJL model at the level of leading approximation for quark propagator and two-particle amplitude.

JafarovR.G.. *Vestnik BGU. No4:143, 2004.*

$c(MeV)$	$m(MeV)$	ξ	$\kappa = 3gm^2/2\pi^2$
-210	357	0.333	0.373
-220	356	0.289	0.322
-230	354	0.252	0.277
-240	353	0.221	0.242
-250	352	0.195	0.212

Таблица: 1. The model parameters in leading order (dimensionally-analytical regularization): chiral condensate c, quark mass m, regularization parameter ξ and dimensionless coupling κ.

Subsection 4.3.1. Model parameters

$c(MeV)$	$m(MeV)$	$\Lambda(MeV)$	$\kappa_\Lambda = 3g\Lambda^2/2\pi^2$
-210	423	733	1.86
-220	323	791	1.488
-230	276	873	1.315
-240	253	947	1.240
-250	236	1029	1.187

Таблица: 2. The model parameters in leading order (4-dimensional cutoff): chiral condensate c, quark mass m, regularization parameter Λ and dimensionless coupling κ_Λ.

Subsection 4.4. Meson amplitudes contributions in chiral condensate and in quark propagator

Since the foundation of the NJL model is non-renormalizable interaction, the quit essential point of the model is a regularization. It already advances in the literature an opinion, that the NJL model for different regularization can lead to different physical results.
First-order equations of the MFE define corrections to quark propagator. First-order mass operator $\Sigma^{(1)} = S_0^{(-1)} \star S^{(1)} \star S_0^{(-1)}$, where $S^{(1)}$ is a first-order correction to quark propagator, is defined by equation

$$\Sigma^{(1)} = S_0(x)A_\sigma(x) + 3S_0(-x)A_\pi(x) + igtrS^{(1)}(0) \qquad (24)$$

Let us to introducing dimensionless NLO mass functions $a^{(1)}$ and $b^{(1)}$:

$$\Sigma^{(1)} \equiv a^{(1)}\hat{p} - b^{(1)}m. \qquad (25)$$

and using the formula (23) for ratio of NLO condensate to the LO condensate, we obtain from (24) the expressions for $a^{(1)}$ and $b^{(1)}$ in momentum space the following formulaes:

$$p^2 a^{(1)}(p^2) = \int d\tilde{q} \frac{p^2 - (pq)}{m^2 - (p-q)^2} [A_\sigma(q) - 3A_\pi(q)], \tag{26}$$

$$b^{(1)}(p^2) = r - \int d\tilde{q} \frac{1}{m^2 - (p-q)^2} [A_\sigma(q) + 3A_\pi(q)]. \tag{27}$$

Using expression (23) for the NLO mass operator, we may to rewrite inverse quark propagator

$$S^{-1} = m - \hat{p} - \Sigma^{(1)} \tag{28}$$

as the form:

$$S^{-1} = (1 + b^{(1)})m - (1 + a(1))\hat{p} \tag{29}$$

(where, according gap equation, m is LO quark mass).

Subsection 4.4.

Suppose the propagator has a pole in point $p^2 = m_r^2$, which corresponds to a particle with mass m_r, which corresponds to a particle with mass m_r:

$$b(m_r^2) = m_r a(m_r^2)$$

Since $a^{(1)}$ and $b^{(1)}$ are small additions ($a^{(1)} \ll 1$, $b^{(1)} \ll 1$),
we can to expand $a^{(1)}(m_r^{(2)})$ and $b^{(1)}(m_r^{(2)})$ near the point m and to obtain the formula for the quark-mass correction $\delta m \equiv m_r - m$:

$$\frac{\delta m}{m} \cong b^{(1)}(m^{(2)}) - a^{(1)}(m^{(2)}). \tag{30}$$

Jafarov R.G.:*Izv. vuzov Fiz. No7:31, 2006.*

Subsection 4.4.1. Sigma-meson amplitudes contribution

Consider a contribution of scalar amplitude in pole approximation A_σ in quark mass. In correspondence with Eqs. (26) and (27) we have

$$p^2 a_\sigma^{(1)}(p^2) = \int d\tilde{q} \frac{p^2 - (pq)}{m^2 - (p-q)^2} A_\sigma(q), \qquad (31)$$

$$b_\sigma^{(1)}(p^2) = r_\sigma - \int d\tilde{q} \frac{1}{m^2 - (p-q)^2} A_\sigma(q). \qquad (32)$$

To calculate this contributions we use the leading-singularity approximation for amplitudes:

$$A_\sigma^{pole} = \frac{1}{4n_c I_0(4m^2)(4m^2 - p^2)|_{p \to 4m^2}}, \qquad (33)$$

with

$$[I_0(4m^2)]^{FDC} = \frac{i}{(4\pi)^2}[\log(1+x) - \sqrt{x}\arctan\frac{1}{\sqrt{x}}] -$$

according Eq. (22) in $p^2 = 4m^2$, for FDG regularization and Eq. (21) for DAR.

Subsection 4.4.1. Sigma-meson amplitudes contribution

From Eq. (23) we obtain the quantity r_σ in DAR. A computation gives us the following values for sigma-meson contribution:
$\xi = 0.25$ we obtain $(r_\sigma)^{DAR} = -0.033$;
$\xi = 0.4$ we obtain $(r_\sigma)^{DAR} = -0.01$.
The sigma-meson contribution is small in comparison of the contribution and possesses the opposite sign, i.e. it decrease the common contribution. For FDC regularization the leading-singularity approximation for A_σ coincides with the pole approximation. This quantity for FDC is a function of $x \equiv \Lambda^2/m^2$. The values of $r_\sigma^{FDC}(x)$ for two characteristic values of ratio:
$x = 3\ (c^{(0)} = -210 MeV)$ - $r_\sigma^{FDC}(3) = -0.007$;
$x = 19\ (c^{(0)} = -250 MeV)$ - $r_\sigma^{FDC}(19) = -0.116$.
In contrast to the DAR, the sign of sigma contribution for FDC is the same as for pion contribution.

Subsection 4.4.1. Sigma-meson amplitudes contribution

A sigma-correction to quark mass for DAR given by formula

$$\left(\frac{\delta m_\sigma}{m}\right)^{DAR} = r_\sigma^{DAR} - \frac{\cos \pi\xi}{4^{1+\xi} n_c \pi (1/2 - \xi)}$$

and attains:
at $\xi = 0.25$: $\delta m_\sigma^{DAR} = -0.086m$,
at $\xi = 0.4$: $\delta m_\sigma^{DAR} = -0.056m$.

$$\left(\frac{\delta m_\sigma}{m}\right)^{FDC} = r_\sigma^{FDC} - \frac{4\log(1+x/4) - \log(1+x)}{8n_c[\log(1+x) + \sqrt{x}\arctan\sqrt{1/x}]},$$

at $x = 3$: $\delta m_\sigma^{FDC} = -0.022m$;
at $x = 19$: $\delta m_\sigma^{FDC} = -0.158m$.
Since a pion correction to quark mass in both regularizations equal zero (see below), these values are full corrections to quark mass.

Pole approximation:

Pseudoscalar amplitude A_π naturally is associated with the pion, which in the chiral limit is a massless Goldstone particle. In both regularizations under consideration we can define a pion propagator as a pole term of A_π^{pole}, which leads to the singularity of pseudoscalar amplitude and in both regularization are as following:

$$\left(A_\pi^{pole}\right)^{DAR} = \frac{1}{12p^2 I_0^{DAR}(0)} = -\frac{2igm^2}{\xi p^2}, \tag{34}$$

$$\left(A_\pi^{pole}\right)^{FDC} = \frac{1}{12p^2 I_0^{FDC}(0)} =$$

$$= -i\frac{4\pi^2}{3\left(\log(1+x) - \frac{x}{1+x}\right)p^2}. \tag{35}$$

Subsection 4.4.2. Pion amplitudes contribution

The pion contribution NLO condensate in pole approximation of pion amplitude in both regularizations (DAR Eq.(34) and FDC regularization Eq.(35)) is calculated by Eq. in pion channel

$$r = \frac{24ign_c}{1 - 8ign_c J} \int d\tilde{p} d\tilde{q} \frac{[m^2 - p^2 + 2(pq)] A_\pi(q)}{(m^2 - p^2)^2 [m^2 - (p-q)^2]},$$

where J is 2-loop integral. All integrals over dp and dq can be calculated in closed form, and the results in both regularizations are the very simple expressions:

$$r_\pi^{DAR} = \frac{1}{8\xi}, \tag{36}$$

$$r_\pi^{FDC} = -\frac{\log(1+x)}{8\left(\log(1+x) - \frac{x}{1+x}\right)}, \tag{37}$$

Jafarov R.G.:*Fizika Azerb. NAS, No1:27, 2006(arXiv:hep-ph/0412114).*

Subsection 4.4.2. Pion amplitudes contribution

According the Eqs. (26)-(27) the NLO mass functions $a^{(1)}$ and $b^{(1)}$ in pion channel are defined by the following equations:

$$p^2 a_\pi^{(1)}\left(p^2\right) = -3\int \frac{d\widetilde{q}}{m^2-(p-q)^2} A_\pi^{pole}(q), \tag{38}$$

$$b_\pi^{(1)}\left(p^2\right) = r_\pi - 3\int \frac{d\widetilde{q}}{m^2-(p-q)^2} A_\pi^{pole}(q) \tag{39}$$

Using the leading singularity approximation for $\left(A_\pi^{pole}\right)^{DAR}$ (34) and $\left(A_\pi^{pole}\right)^{FDC}$ (35) in (38) and (39) after calculating the integrals in DAR and FDC regularization we obtain for the pion corrections to the quark mass in next expressions according to (30)

$$(\delta m_\pi/m)^{DAR} = r_\pi^{DAR} - 1/8\xi, \tag{40}$$

$$(\delta m_\pi/m)^{FDC} = r_\pi^{FDC} + \log(1+x)/8(\log(1+x) - x/(1+x)) \tag{41}$$

The pion contribution in quark mass is equal zero, according to (36) and (37).

However, since the model is not renormalizable in four space-time dimensions, the physical results and parameters depend on the regularization method. This lead us to calculate the correction to quark mass beyond the non-pole approach of the amplitude. Using the expressions of pion amplitude (19) and the integral (21) in DAR, we can to calculate the ratio in pion sector r_π. Also, having calculated in DAR NLO mass functions by the Eqs. (38) and (39), according the formula (30) of NLO quark mass correction we obtain:

$$\left[\left(\frac{\delta m_\pi}{m}\right)^{DAR}\right]^{non-pole} = \left[r_\pi^{DAR}\right]^{non-pole} -$$

$$\frac{1}{2\sqrt{\pi}\Gamma(2-\xi)}\sum_{k=0}^{\infty} 4^k \Gamma(1-\xi-k)\Gamma(3/2-k)\times$$

$$F(1+\xi+k, 1+k; 2-\xi; 1),$$

where

$$\left[r_\pi^{DAR}\right]^{non-pole} = -2\frac{\sin(\pi\xi)}{\pi\xi}\int_0^\infty dz \frac{z^{-1-\xi}}{F(1+\xi,1;3/2;-z/4)}\times$$

$$\int_0^1 du \frac{1-u}{[1+u(1-u)z]^{1+\xi}}\cdot\left[1-\xi+(1+\xi)\frac{1+u(u-2)z}{1+u(1-u)z}\right]$$

From here is clear, that the result differ from zero.
This means, that the zero value of the pion correction to quark mass is independent from regularization choice in NJL model in leading singularity approach of pseudoscalar amplitude.

R.G. Jafarov: *Georgian El. Science Jour. Physics No2:13, 2009.*

Subsection 4.4.3. Improved model parameters

The condensate and the quark-mass corrections allow us to specify parameters of the $SU(2)-$NJL model.
We modify a formula for the condensate as follows:

$$c^3 = c_0^3 + c_1^3 = -\frac{m}{2g}(1+r). \tag{42}$$

The formula for f_π stays the same, since corrections to amplitudes generate in the next(second) order of MFE. The quark mass is the mass m_r:

$$m_r = m + \delta m.$$

Values of the model parameters for this improved choice are given in Tables3. and 4.

R.G. Jafarov: *Izv. Vuzov. Fizika. No7:31, 2006.*

Subsection 4.4.3. Improved model parameters. DAR

$$m = -\frac{c^3}{f_\pi^2} \cdot \frac{\xi}{1 + \frac{1}{8\xi}},$$

$$1 = \frac{3}{4\pi^2} \cdot \frac{m^2}{f_\pi^2} \Gamma(1 + \xi),$$

$c(MeV)$	$m(MeV)$	ξ	$\kappa = 3gm^2/2\pi^2$
-210	339	0.432	0.486
-220	336	0.385	0.434
-230	333	0.346	0.387
-240	330	0.312	0.334
-250	328	0.284	0.316

Таблица: 3. The model parameters with first-order (dimensionally-analytical regularization): chiral condensate c, quark mass m_r, regularization parameter ξ and dimensionless coupling κ.

Subsection 4.4.3. Improved model parameters. 4D cutoff

$$m = -\frac{c^3}{f_\pi^2} \cdot \frac{log(1+x) - \frac{x}{1+x}}{x - log(1+x)}(1 + r(x)),$$

$$1 = \frac{3}{4\pi^2} \cdot \frac{m^2}{f_\pi^2}[log(1+x) - \frac{x}{1+x}],$$

$c(MeV)$	$m(MeV)$	$\Lambda(MeV)$	$\kappa_\Lambda = 3g\Lambda^2/2\pi^2$
-240	310	785	1.501
-250	283	819	1.408

Таблица: 4. The model parameters in leading order (4D cutoff): chiral condensate c, quark mass m, regularization parameter Λ and κ_Λ.

Subsection 4.4.3. Improved model parameters. 4D cutoff

Table 4. does not contain the parameter values at $c = -210MeV$, $c = -220MeV$ and $c = -230MeV$. These values are absent due to following reason: the system of equations , which determines these parameters, has no solution at $f_\pi = 93MeV$ and at $|c| \leq 230MeV$. There is very important circumstance - for 4-dimensional cutoff the meson contributions can destabilize the NJL model. Though these contributions are relatively small (they do not exceed $25perecentege$ from the leading contribution), but their opposite sign leads to a non stability of all the system. The situation is very similar to that of pointed of work by Kleinert at all.

Subsection 4.4.3. Improved model parameters

Note, that increasing a number of flavors, i.e. $U(N_f)$-NJL model, the situation takes a turn for the worse, because a main pseudo-scalar contribution is proportional to N_f.
At that for DAR the situation is principally different: due to the coincidence of sign of the meson contributions with the sign of leading contribution in condensate for this regularization a stabilization of the model takes place. It is clearly seen, values of regularization parameter ξ increase in comparison with corresponding leader-order values, i.e. shift to a region of stability of model, where these meson contributions decrease.

R.G. Jafarov and V.E. Rochev: *Izv. Vuzov. Fizika. No4:20, 2006 (arXiv: hep-ph/0406333).*

The second step contains the equations for the four S_4- and three-particle S_3 functions and also the equations for the two-particle function $S_2^{(1)}$ and the second-order corrections to the quark propagator $S^{(2)}$. For these four functions we have a system of four integral equations. All these equations (and all equations of following steps of the iteration scheme) possess the structure, which is similar to the structure of Eq. (14):

$$S_n = S_n^0 + ig\Big\{(S \cdot S) \star tr[S_n] - (S\gamma_5\tau^a \cdot S) \star tr[\gamma_5\tau^a S_n]\Big\}, \qquad (43)$$

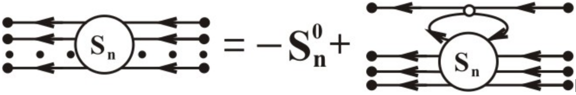


Figure 14.

and differ from each other by the structure of inhomogeneous terms S_n^0.

Section 5. Second step equations

The inhomogeneous term in the equation for four-quark function S_4 is

$$S_4^0 = -3 \cdot \Big\{ S \cdot S \cdot S_2 \Big\}, \tag{44}$$

where S_2 is defined in preceding section by Eq. (14).

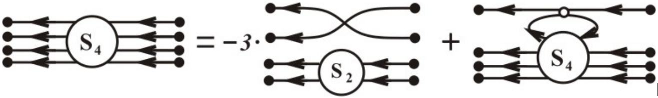


Figure 15.

The inhomogeneous term in the equation for three-quark function S_3

$$S_3^0 = -2\left\{S \cdot S \cdot S^{(1)}\right\} - 2 \cdot \left[S \cdot S_2\right] + ig \cdot S \star \left\{tr[S_4] - \gamma_5 \tau tr[\gamma_5 \tau S_4]\right\}. \quad (45)$$

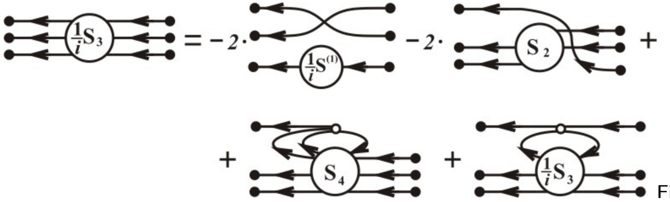


Figure 16.

Section 5. Second step equations

The solution of four-quark equation is the sum of products of two-quark functions S_2:

$$S_4 = 3 \cdot \left\{ S_2 \cdot S_2 \right\} \tag{46}$$

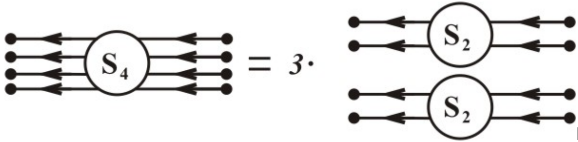


Figure 17.

R.G. Jafarov and V.E. Rochev: *Proceedings of the XXVIII International Workshop on the FPHEP and Field Theory(2005), New Physics at Colliders and Cosmic Rays, Moscow Region, Protvino, p.27-33, 2005 and in Proceedings of Workshop LHP06, Tehran, Iran, 2006 (arXiv: hep-ph/0609183).*

Section 6. Structure of third step of iteration step equations

As we have showed above the equation for the four-quark function S_4 has a simple exact solution which is the product of first-order two-quark functions (see Eq. (46)). As it seen from this solution, the pion-pion scattering in NJL model is suppressed, i.e. in the second order of MFE this scattering is absent. This process arises in the third order of our iterative scheme, i.e. in NLO four-quark function $S_4^{(1)}$.
The third-step generating functional is

$$G^{(3)}[\eta] = \left\{ \frac{1}{6!}\mathrm{Tr}\Big(S_6 * \eta^6\Big) + \frac{1}{5!}\mathrm{Tr}\Big(S_5 * \eta^5\Big) + \frac{1}{4!}\mathrm{Tr}\Big(S_4^{(1)} * \eta^4\Big) + \right.$$

$$\left. \frac{1}{3!}\mathrm{Tr}\Big(S_3^{(1)} * \eta^3\Big) + \frac{1}{2}\mathrm{Tr}\Big(S_2^{(2)} * \eta^2\Big) + \mathrm{Tr}\Big(S^{(3)} * \eta\Big) \right\} G^{(0)}.$$

R.G. Jafarov: *Fizika Azerb NAS, XI, No 3:27,2005.*

Section 6. Structure of third step of iteration step equations

After standard operations we obtain the equations for six-quark function S_6 and for five-quark function S_5. Inhomogeneous terms are following:

$$S_6^0 = 5 \cdot \left\{ -S \cdot S \cdot S_4 \right\} \tag{47}$$

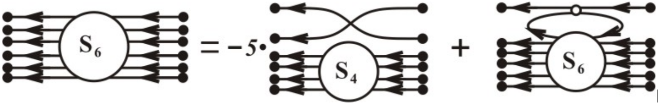


Figure 18.

and

$$S_5^0 = -4\cdot\left\{S\cdot S\cdot S_3\cdot\right\} - 4\cdot\left[S\cdot S_4\right] + ig\left\{tr\left[S\star S_6\right] - tr\left[S\gamma_5\tau^a\star S_6\gamma_5\tau^a\right]\right\}, \tag{48}$$

accordingly. The equations for six-quark function and for the five-quark function with inhomogeneous term (47) and (48) in our iteration scheme are new. The third step of iterative scheme gives us the equation for four-quark function ($S_4^{(1)}$).

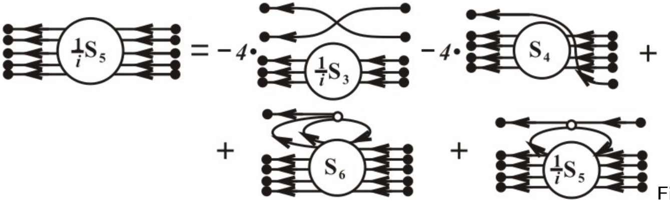


Figure 19.

Section 6. Structure of third step of iteration step equations

As we note above the structure of this equation have are the form (43) with following inhomogeneous term

$$(S_4^{(1)})^0 = -3\cdot\left\{S\cdot S\cdot S_2^{(1)}\right\} - 3\cdot\left[S\cdot S_3\right] + ig\left\{tr\left[S\star S_5\right] - tr\left[S\gamma_5\tau^a\star S_5\gamma_5\tau^a\right]\right\}. \tag{49}$$

The equation for NLO four-quark function $S_4^{(1)}$ gives us possibility to describe the pion-pion scattering in quark fields context. The inhomogeneous term (49) of equations for four-quark function $S_4^{(1)}$ contains five-quark function S_5, three-quark function S_3 and two-quark function $S_2^{(1)}$. The inhomogeneous term (48) for five-quark equation include the six-quark function S_6, four-quark function S_4 and three-quark function S_3. Before the investigation of four-quark function $S_4^{(1)}$ it is necessary to find the solution of equation for six-quark function S_6, because the inhomogeneous part (48) includes function S_6. Also it is necessary to find a solution of equation for NLO two-quark function $S_2^{(1)}$.

Section 6. Structure of third step of iteration step equations

The solution of six-quark equation is the sum of products of two-quark functions S_2 and four-quark functions S_4:

$$S_6 = 5 \cdot \left\{ S_2 \cdot S_4 \right\} \tag{50}$$

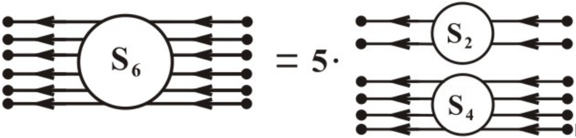


Figure 20.

In this step we obtain also the equations for NLO three-quark function $S_3^{(1)}$, NNLO two-quark function $S_2^{(2)}$ and the equation for NNNLO correction to the quark propagator $S^{(3)}$, which matter the forms (43), at $n = 3$, $n = 2$, $n = 1$, accordingly.

Jafarov R.G.:*Izv. Akad. nauk Azerb. v.XXVI, No2:3, 2006.*

Section 7. The formalism of other type sources

In this last Section we consider the generalization of MFE of Section 2, which includes other types of multi-quark sources except of bilocal source η. Such generalization can be useful for the description of baryons in the framework of MFE.

R.G. Jafarov and V.E. Rochev: *Talk given in QUARKS-2010 16th International Seminar on High Energy Physics Kolomna, Russia, 6-12 June, 2010.*

Subsection 7.1. The formalism with diquark sources

Firstly, consider the formalism with diquark sources. For this purpose, we add two diquark-source terms ξ and $\bar{\xi}$ in the exponent of Eq. (11) for generating functional G:

$$G(\eta,\xi,\bar{\xi}) = \int D(\psi,\bar{\psi}) \exp i\Big\{ \int dx L - \int dx dy \bar{\psi}(y)\eta(y,x)\psi(x) + \\ + \int dx_1 dx_2 \bar{\psi}(x_1)\bar{\psi}(x_2)\xi(x_1,x_2) + \int dx_1 dx_2 \bar{\xi}(x_1,x_2)\psi(x_1)\psi(x_2)\Big\}. \quad (51)$$

With these sources SDE (10) is modified as follows:

$$G + i\hat{\partial}\frac{\delta G}{\delta\eta} + ig\Big\{\frac{\delta}{\delta\eta} tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a \frac{\delta}{\delta\eta} tr\left[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\Big\} = \\ = \eta\star\frac{\delta G}{\delta\eta} + 2\cdot\frac{\delta G}{\delta\xi}\star\xi. \quad (52)$$

Subsection 7.1. The formalism with diquark sources

We have, apart from SDE (52), the additional SDE, which generates by new sources:

$$i\hat{\partial}\frac{\delta G}{\delta\bar{\xi}} + ig\Big\{\frac{\delta}{\delta\bar{\xi}}tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\bar{\xi}}tr\left[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\Big\} =$$

$$= \eta\star\frac{\delta G}{\delta\bar{\xi}} - 2\cdot\bar{\xi}\star\frac{\delta G}{\delta\eta}. \quad (53)$$

It should be noted, that the presence of the new diquark source leads to the connection condition for derivatives of generating functional:

$$\frac{\delta^2 G}{\delta\bar{\xi}(x_2,x_1)\delta\eta(y,x)} = -\frac{\delta^2 G}{\delta\bar{\xi}(x_1,x)\delta\eta(y,x_2)}. \quad (54)$$

Due to this connection condition SDE (53) can be rewritten in the alternative forms. These alternative forms, being fully equivalent from the point of view of an exact solution of SDE's, can lead to different approximations in the MFE. The choice of the suitable forms for the construction of MFE in the case should be made with an assistance of corresponding physical reasons.

Subsection 7.2. The formalism with triple-sources

In the very similar manner one can introduce three-quark, or baryon sources. These sources can be used for the direct description of nucleons and other baryons omitting the intermediate diquark modelling. The generating functional with anti-commutative three-quark sources ζ and $\bar{\zeta}$ is

$$G(\eta,\zeta,\bar{\zeta}) = \int D(\psi,\bar{\psi}) \exp i\Big\{ \int dx L - \int dx dy \bar{\psi}(y)\eta(y,x)\psi(x) +$$

$$+ \int dx_1 dx_2 dx_3 \bar{\psi}(x_1)\bar{\psi}(x_2)\bar{\psi}(x_3)\zeta(x_1,x_2,x_3) +$$

$$+ \int dx_1 dx_2 dx_3 \bar{\zeta}(x_1,x_2,x_3)\psi(x_1)\psi(x_2)\psi(x_3)\Big\}. \tag{55}$$

Subsection 7.2. The formalism with triple-sources

The master-equations for generating of SDEs are follows

$$0=\int D(\psi,\bar{\psi})\frac{\delta}{\delta\bar{\psi}^{\alpha}(x)_{j}^{c}}\bar{\psi}^{\beta}(y)_{k}^{d}\times$$

$$\times\exp i\Big[\int dxL-\int dxdy\bar{\psi}(y)\eta(y,x)\psi(x)+$$

$$+\int dxdydz\bar{\xi}(xyz)\psi(x)\psi(y)\psi(z)+\int dxdydz\bar{\psi}(x)\bar{\psi}(y)\bar{\psi}(z)\xi(xyz)\Big],$$

$$0=\int D(\psi,\bar{\psi})\frac{\delta}{\delta\bar{\psi}^{\alpha_1}(x_1)_{j_1}^{c_1}}\psi^{\alpha_3}(x_3)_{j_3}^{c_3}\psi^{\alpha_2}(x_2)_{j_2}^{c_2}\times$$

$$\times\exp i\Big[\int dxL-\int dxdy\bar{\psi}(y)\eta(y,x)\psi(x)+$$

$$+\int dxdydz\bar{\xi}(xyz)\psi(x)\psi(y)\psi(z)+\int dxdydz\bar{\psi}(x)\bar{\psi}(y)\bar{\psi}(z)\xi(xyz)\Big].$$

i.e.

$$\bar{\psi}^{\beta}(y)^{d}_{k}\,\psi^{\alpha}(x)^{c}_{j} \to i\frac{\delta}{\delta\eta^{\beta\alpha}(y,x)^{dc}_{kj}},$$

$$\psi^{\gamma}(z)^{e}_{l}\psi^{\beta}(y)^{d}_{k}\,\psi^{\alpha}(x)^{c}_{j} \to -i\frac{\delta}{\delta\bar{\xi}^{\gamma\beta\alpha}(zyx)^{edc}_{lkj}},$$

$$\bar{\psi}^{\gamma}(z)^{e}_{l}\bar{\psi}^{\beta}(y)^{d}_{k}\,\bar{\psi}^{\alpha}(x)^{c}_{j} \to i\frac{\delta}{\delta\xi^{\gamma\beta\alpha}(zyx)^{edc}_{lkj}}.$$

SDE (10) with three-quark sources is modified as follows:

$$G + i\hat{\partial}\frac{\delta G}{\delta\eta} + ig\Big\{\frac{\delta}{\delta\eta}tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\eta}tr\left[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\Big\} =$$

$$= \eta\star\frac{\delta G}{\delta\eta} - 3\cdot\frac{\delta G}{\delta\xi}\star\zeta. \qquad (56)$$

As above, apart from SDE (56), the additional SDE exists, which generates by the three-quark sources:

$$i\hat{\partial}\frac{\delta G}{\delta\bar{\zeta}} + ig\Big\{\frac{\delta}{\delta\bar{\zeta}}tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\bar{\zeta}}tr\left[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\Big\} =$$

$$= \eta\star\frac{\delta G}{\delta\bar{\zeta}} + 3i\cdot\frac{\delta^2 G}{\delta\eta\delta\eta}\bar{\zeta}. \qquad (57)$$

The connection condition for the derivatives of the generating functional, which is very similar to the condition(54), also exists in the three-quark-source formalism, and also leads to alternative forms of SDE(57).

Section 7

The method of the construction of MFE for these system of equations is similar to that of Section 2.
An analysis of this construction is the object of future investigations!.

Subsection 7.3. Bethe-Salpeter type equation for three-quark Green function

Leading order:

$$G_0+(i\hat{\partial}-m_0)\frac{\delta G_0}{\delta\eta}+ig\left\{\frac{\delta}{\delta\eta}tr\left[\frac{\delta G_0}{\delta\eta}\right]+i\gamma_5\tau^a\frac{\delta}{\delta\eta}tr\left[i\gamma_5\cdot\tau^a\cdot\frac{\delta G_0}{\delta\eta}\right]\right\}=0,$$

$$(i\hat{\partial}-m_0)\frac{\delta G_0}{\delta\bar{\xi}}+ig\{\frac{\delta}{\delta\bar{\xi}}tr[\frac{\delta G_0}{\delta\eta}]+(i\gamma_5\tau^a)_1\frac{\delta}{\delta\bar{\xi}}tr[i\gamma_5\tau^a\frac{\delta G_0}{\delta\eta}]\}=0. \quad (58)$$

$$G_0=\exp Tr(S\star\eta)$$

Leading-order propagator:

$$S^{-1}=(m_0-i\hat{\partial}-igtrS)$$

Subsection 7.3. Bethe-Salpeter type equation for three-quark Green function

First step:

$$G_1 = P_1 G_0,$$

$$P_1 = \frac{1}{2} S_2 \eta^2 + S^{(1)} \eta + \bar{\xi} G_3^{(0)} \xi$$

S_2 is two-particle function. $G_3^{(0)}$-eq.:

$$(i\hat{\partial} - m_0) G_3^{(0)} \xi + ig G_3^{(0)} \xi \cdot tr S = 3iS \cdot S \star \xi.$$

Solution:

$$G_3^{(0)} = -3iS \cdot S \cdot S$$

Subsection 7.3. Bethe-Salpeter type equation for three-quark Green function

Second step:

$$G_2 = P_2 G_0,$$

$$P_2 = \frac{1}{4!} S_4 \eta^4 + \frac{1}{3!} S_3 \eta^3 + \frac{1}{2} S_2^{(1)} \eta^2 + S^{(2)} \eta +$$

$$\frac{1}{2} \bar{\xi} H_5 \eta^2 \xi + \bar{\xi} H_4 \eta \xi + \frac{1}{2} \bar{\xi}^2 G_6 \xi^2 + \bar{\xi} G_3^{(1)} \xi$$

From SDE (58):

$$-S^{-1}[\frac{1}{2} H_5 \eta^2 \xi + H_4 \eta \xi + \bar{\xi} G_6 \xi^2 + G_3^{(1)} \xi] + ig[tr H_5 \eta + i\gamma_5 \tau^a tr i\gamma_5 \tau^a H_5 \eta] \xi +$$

$$+ig[tr H_4 + i\gamma_5 \tau^a tr i\gamma_5 \tau^a H_4] \xi =$$

$$= \eta \star G_3^{(1)} \xi + 3i\{S_2 + [S_2 \eta + S^{(1)}] \cdot S + S \cdot [S_2 \eta + S^{(1)}] +$$

$$+S \cdot S[\frac{1}{2} S_2 \eta^2 + S^{(1)} \eta + \bar{\xi} G_3^{(1)} \xi]\} \cdot \xi.$$

Subsection 7.3. Bethe-Salpeter type equation for three-quark Green function

O($\bar{\xi}\xi^2$):

$$G_6 = -3iS \cdot S \cdot S \cdot G_3^{(0)}$$

O($\eta^2\xi$):

$$H_5 = -3iS \cdot S \cdot S \cdot S_2$$

O($\eta\xi$):

$$H_4 = -S \cdot G_3^{(0)} - 3i[S \cdot S_2 \cdot S + S \cdot S \cdot S_2 + S \cdot S \cdot S \cdot S^{(1)}] +$$
$$+ig[S \cdot trH_5 + S \cdot i\gamma_5\tau^a \cdot tri\gamma_5\tau^a H_5]$$

O(ξ):

$$G_3^{(1)} = -3i[S \cdot S_2 + S \cdot S \cdot S^{(1)} + S \cdot S^{(1)} \cdot S] +$$
$$+ig[S \cdot trH_4 + S \cdot i\gamma_5\tau^a \cdot tri\gamma_5\tau^a H_4]$$

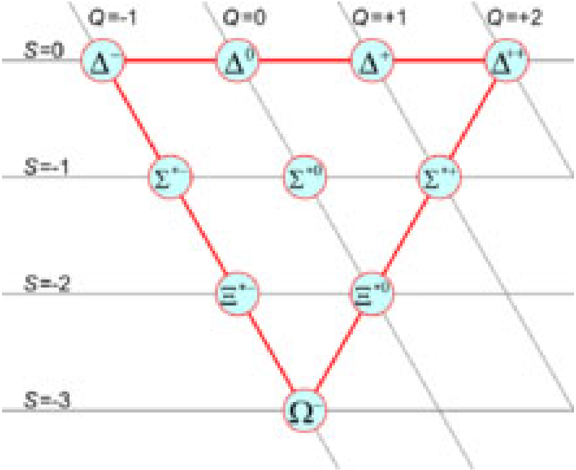
Subsection 7.3. Bethe-Salpeter type equation for three-quark Green function

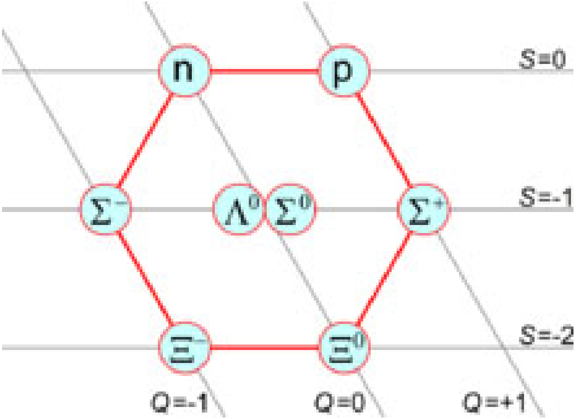
Consequently,

$$G_3 = G_3^{(0)} + G_3^{(1)} =$$

$$-3i[S{\cdot}S{\cdot}S + S{\cdot}S_2 + S{\cdot}S{\cdot}S^{(1)} + S{\cdot}S^{(1)}{\cdot}S] + ig[S{\cdot}tr H_4 + S{\cdot}i\gamma_5\tau^a{\cdot}tr i\gamma_5\tau^a H_4].$$

Subsection 7.3. A possible application of this BSE




n
p
S=0
Σ⁻
Λ⁰
Σ⁰
Σ⁺
S=-1
Ξ⁻
Ξ⁰
S=-2
Q=-1
Q=0
Q=+1

Acknowledgments

Thanks for patience and attention!

Thanks for questions!

Much to my regret!

HAPPY NEW YEAR!

to encounter