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A NEW METHOD FOR RANKING FUZZY NUMBERS WITHOUT CONCERNING OF REAL NUMBERS

R. EZZATI[1], R. ENAYATI[1], A. MOTTAGHI[1], R. SANEIFARD[2]

Abstract. In this paper, a new ranking function is proposed to compare m fuzzy quantities in $m-1$ steps. The ranking function is in terms of the core and margin and $\alpha - cuts$ of fuzzy quantity, which are considered simultaneously. Also, in our procedure the real numbers will be omitted. So, we have illustrated several typical examples to compare the proposed method with some other ranking methods.

Keywords: fuzzy numbers, ranking, the total difference.

AMS Subject Classification: 03B52.

1. Introduction

In a fuzzy environment, ranking fuzzy numbers is a very important decision making procedure. Many authors have investigated various ranking methods. Wang and Kerre [23, 24] organized more than thirty ordering indices and classified them into three categories, fuzzy mean and spread, fuzzy scoring and preference relation. In the first class, each index is associated with a mapping from the set of fuzzy quantities to the real line in order to transform the involved fuzzy quantities into real numbers. Fuzzy quantities are then compared according to the order of corresponding real numbers. And it has only been referenced to the first article of these authors. Other contributions in this field are: Ranking fuzzy numbers based on fuzzy simulation analysis method proposed by Huijun Sun and Jianjun Wu [22], The revised method of ranking fuzzy numbers with an area between the centroid and original points proposed by Yu-Jie Wang and Hsuan-Shih Lee [27], Ranking fuzzy numbers by distance minimization proposed by B. Asady and A. Zendehnam [3], Ranking fuzzy numbers with preference weighting function expectations proposed by Xin-Wang Liu and Shi-Lian Han [20], Ranking fuzzy numbers with an area method using radius of gyration proposed by Yong Deng, Zhu Zhenfu and Liu Qi [15], An approximate approach for ranking fuzzy numbers based on left and right dominance proposed by Liang-Hsuan Chen and Hai-Wen Lu [7], The revised method of ranking LR fuzzy number based on deviation degree proposed by B. Asady [4], A new approach for ranking of trapezoidal by fuzzy numbers proposed by S. Abbasbandy and T. Hajjari [1], Ranking LR fuzzy number based on deviation degree proposed by Zhong-Xing Wanga, Yong-Jun Liua, Zhi-Ping Fanb and Bo Feng [21], Area ranking of fuzzy numbers based on positive and negative ideal points proposed by Ying-Ming Wanga and Ying Luob [25], Ranking of fuzzy numbers by sign distance proposed By S. Abbasbandy and B. Asady [2].

[1] Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran,
e-mail: ezati@kiau.ac.ir
[2] Department of Mathematics, Oroumieh Branch, Islamic Azad University, Oroumieh, Iran
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Bortolan and Degani [6] were the first authors who made a review of the literature and provided not only a systematical review of fuzzy ranking research but also gave results of comparisons among various ranking methods, checking the disparity of results among them. In the Bortolan and Degani's paper, there are examples which have been indicated in the paper like Fig. 3. Set 3.

Chen and Hwang [8] were the first author, who presented twenty ranking methods classified into three major classes according to the media that each method uses. There are preference relation method, a fuzzy mean and spread method, fuzzy scoring method, and linguistic method.

So Liou and Wang [19] proposed a method for ranking fuzzy numbers with an integral value that represents a "mean value", and there are examples such as those used in Fig. 5. Set 2. Also This method widely used the method in literature which were modified by Garcia-Cascales and Lamata [16] is able to discriminate in cases such as Fig. 4. Set 1 or Fig. 8. Set 3, while many other methods were not capable. It is very interesting that our method is able to discriminate in cases such as Fig. 7. Set 2 while no method was able to discriminate different types of fuzzy numbers as in Fig. 6. Set 1.

In proposed method, the core and margin and $\alpha - cuts$ are considered according to each of fuzzy quantities simultaneously. Also there are some properties which are useful in ranking a large quantity of fuzzy numbers in this described method. It is clear that the proposed method, with respect to these properties, is more reliable and computationally easier than some methods which were proposed up to now and overcome the shortcoming of the previous methods. For example, comparing the proposed method with the existing method Liang-Hsuan Chen and Hai-Wen Lu [7] reveals that the proposed method is more simple and consistent than the proposed method in [7].

The rest of this paper is organized as follows. Section 2 briefly introduces the fuzzy numbers. Section 3 introduces the ranking approach. Section 4 describes some useful properties. Concluding remarks are finally made in Section 5.

2. Basic definitions and notations

A real fuzzy number can be defined as a fuzzy subset of the real line $\Re$, which is convex and normal. That is, for a fuzzy number A of $\Re$ defined by membership function $\mu_A(x)$, $x \in \Re$, the following relations exist:

$$\max_x \mu_A(x) = 1, \tag{1}$$

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\mu_A(x_1), \mu_A(x_2)], \tag{2}$$

where $x_1, x_2 \in \Re, \forall \lambda \in [0,1]$. A fuzzy number A with the membership function $\mu_A(x)$, $x \in \Re$, can be defined as follows:

$$\mu_A(x) = \begin{cases} \mu_A^L(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ \mu_A^R(x), & c \leq x \leq d, \\ 0, & otherwise, \end{cases} \tag{3}$$

where $\mu_A^L(x)$ is the left membership function that is an increasing function and $\mu_A^L(x) : [a,b] \to [0,1]$. Meanwhile, $\mu_A^R(x)$ is the right membership function that is a decreasing function and $\mu_A^R(x) : [c,d] \to [0,1]$. In addition, a trapezoidal fuzzy number denoted by $[a,b,c,d]$ can also signify a triangular fuzzy number if $b = c$. Assume that, every fuzzy number is bounded; i.e. $-\infty < a, d < \infty$. The support of fuzzy number A is denoted as $supp(A)$ and obtained by $\{x \in \Re | \mu_A(x) > 0\}$ and the $core(A)$ leads to a set such that $\{x \in \Re | \mu_A(x) = 1\}$.

The $\alpha - cuts$ (level sets) of fuzzy number A can be obtained as follows:

$$A(\alpha) = \{x \in \Re | \mu_A(x) \geq \alpha\}, \alpha \in [0,1], \tag{4}$$

where $A(\alpha)$ is a convex subset that belongs to $\Re$. The lower and upper limits of α-cut A_i are defined as:

$$\underline{A_i}(\alpha) = \inf\{x | \mu_{A_i}(x) \geq \alpha\}, \tag{5}$$

$$\overline{A_i}(\alpha) = \sup\{x | \mu_{A_i}(x) \geq \alpha\}, \tag{6}$$

where $A_i(1) - \underline{A_i}(0)$ and $\overline{A_i}(0) - A_i(1)$ are left and right spreads for each fuzzy number A_i, respectively.

3. The total difference between fuzzy numbers

In this section, an operator will be proposed as $D(A, B)$ and called the total difference between the fuzzy numbers. To compare A and B, we need to determine some parameters as follows:

$$M_A = \sup\{\alpha : \mu_A(x) = \alpha, \ \forall\ x \in supp(A)\}. \tag{7}$$

So $M_A = 1$, if A be a fuzzy number.

$$\overline{core(A)} = \sup\{x : \mu_A(x) = M_A\}, \tag{8}$$

$$\underline{core(A)} = \inf\{x : \mu_A(x) = M_A\}, \tag{9}$$

If A be a fuzzy number such that $supp(A) = [a, b]$, we define:

$$\overline{supp(A)} = \max\{x : x \in supp(A)\} = b, \tag{10}$$

$$\underline{supp(A)} = \min\{x : x \in supp(A)\} = a. \tag{11}$$

So, the total difference of A and B can be organized as:

$$D(A, B) = (\frac{\overline{core(A)} + \underline{core(A)}}{\overline{suppA} - \underline{supp(A)}})^2 \int_0^{M_A} (\overline{A}(r) - \underline{A}(r))\, dr - \tag{12}$$

$$-(\frac{\overline{core(B)} + \underline{core(B)}}{\overline{supp(B)} - \underline{supp(B)}})^2 \int_0^{M_B} (\overline{B}(r) - \underline{B}(r))\, dr.$$

In sum, the algorithm to rank of two fuzzy numbers A and B based on total difference between them is given as follows:

Step 1. Let $supp(A) = [a_1, b_1]$, $supp(B) = [a_2, b_2]$, and $C = min\{a_1, a_2\}$. If C be nonnegative, go to step 2. Otherwise, add the value of $|C|$ to all fuzzy numbers (So, all fuzzy numbers are moved into the nonnegative x-axis), and then go to step 2.

Step 2. Determine the total deference between A and B by applying (3.11), i.e. $D(A, B)$, and then go to step 3.

Step 3. The ranking order is determined based on the following rules:

1) If $D(A, B) > 0$, then $A \succ B$,
2) If $D(A, B) < 0$, then $A \prec B$,
3) If $D(A, B) = 0$ and $\overline{supp(A)} > \overline{supp(B)}$ (i.e. $b_1 > b_2$), then $A \succ B$,
4) If $D(B, A) = 0$ and $\overline{supp(B)} > \overline{supp(A)}$ (i.e. $b_2 > b_1$), then $B \succ A$, otherwise $A \sim B$.

Note that in this method if we encounter fuzzy numbers such that a part of their support or all of their support lies on negative x-axis, we will first obtain minimum of support among all them i.e. k=min$\{\underline{A_i}(0), i = 1, 2, ..., n\}$, and then we will add the absolute value of obtained value to all fuzzy numbers i.e. $A = [a, b, c, d]$ transforms into $A = [a + |k|, b + |k|, c + |k|, d + |k|]$.

4. Numerical examples

In this section, the researchers compare the proposed method with the others in [5, 9, 11, 12, 13, 14, 29].

Example 4.1. Consider the following sets, see Yao and Wu [29].
Set 1: A=(0.4,0.5,1), B=(0.4,0.7,1), C=(0.4,0.9,1).
Set 2: A=(0.3,0.4,0.7,0.9) (trapezoidal fuzzy number) , B=(0.3,0.7,0.9), C=(0.5,0.7,0.9).
Set 3: A=(0.3,0.5,0.8,0.9) (trapezoidal fuzzy number), B=(0.2,0.5,0.9), C=(0.1,0.6,0.8).
To compare other methods, researchers refer reader to Table 1.

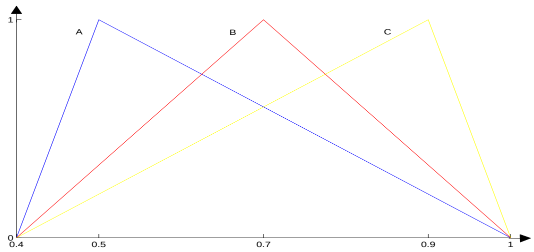

Fig. 1. Set 1.

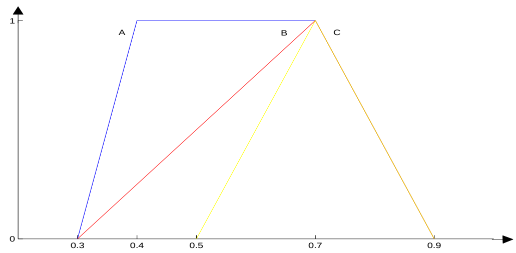

Fig. 2. Set 2.

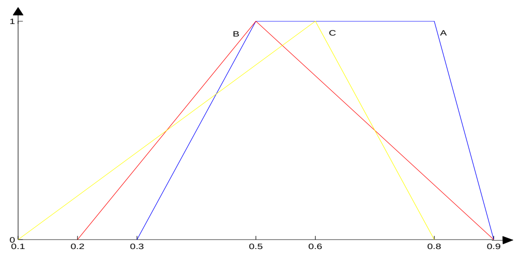

Fig. 3. Set 3.

Table 1. Comparative results of example 4.1.

Authors	Fuzzy number	Set1	Set2	Set3
Proposed method	A, B	-0.8	-0.123	1.401
	B, C	-1.07	-0.815	-0.215
	A, C	-1.87	-0.938	1.716
Results		$A \prec B \prec C$	$A \prec B \prec C$	$B \prec C \prec A$
Sing Distance method	A	1.2000	1.1500	0.0950
with p=1	B	1.4000	1.3000	1.0500
	C	1.6000	1.4000	1.0500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Sing Distance method	A	0.8869	0.8756	0.7853
with p=2	B	1.0194	0.9522	0.7958
	C	1.1605	1.0033	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Distance Minimization	A	0.6	0.575	0.475
	B	0.7	0.65	0.525
	C	0.9	0.7	0.525
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Abbasbandy and Hajjari	A	0.5334	0.5584	0.5250
(Magnitude method)	B	0.7000	0.6334	0.5084
	C	0.8666	0.7000	0.5750
Result		$A \prec B \prec C$	$A \prec B \prec C$	$B \prec A \prec C$
Choobineh and Li	A	0.3333	0.5480	0.5000
	B	0.5000	0.5830	0.5833
	C	0.6670	0.6670	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Yager	A	0.6000	0.5750	0.4500
	B	0.7000	0.6500	0.5250
	C	0.8000	0.7000	0.5500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chen	A	0.3375	0.4315	0.5200
	B	0.5000	0.5625	0.5700
	C	0.6670	0.6250	0.6250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	0.3000	0.2700	0.4000
	B	0.3300	0.2700	0.4200
	C	0.4400	0.3700	0.4200
Results		$A \prec B \prec C$	$A \sim B \prec C$	$A \prec B \sim C$
Chu and Tsao	A	0.2990	0.2847	0.2440
	B	0.3500	0.3247	0.2624
	C	0.3993	0.3500	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$
Yao and Wu	A	0.6000	0.5750	0.4750
	B	0.7000	0.6500	0.5250
	C	0.8000	0.7000	0.5250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Cheng distance	A	0.7900	0.7577	0.7106
	B	0.8602	0.8149	0.7256
	C	0.9268	0.8602	0.7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$

Table 1. (continued)

Authors	Fuzzy number	Set1	Set2	Set3
Cheng CV uniform	A	0.0272	0.0328	0.0693
distribution	B	0.0214	0.0246	0.0385
	C	0.0225	0.0095	0.0433
Results		$A \prec C \prec B$	$A \prec B \prec C$	$A \prec C \prec B$
Cheng CV proportional	A	0.0183	0.0260	0.0471
distribution	B	0.0128	0.0146	0.0236
	C	0.0137	0.0057	0.0255
Results		$A \prec C \prec B$	$A \prec B \prec C$	$A \prec C \prec B$

Note that, in Table 1 and in set 3, for Sign Distance ($p = 1$), Distance Minimization, Chu-Tsao and Yao-Wu methods, the ranking order for fuzzy numbers B and C is $B \sim C$, that looks unreasonable.

Example 4.2. Consider the following sets:

Set 1: $A = (1, 2, 5)$, $B = (0, 3, 4)$ and $C = (2, 2.5, 3)$.

Set 2: $A = (3, 5, 7; 1)$, $B = (3, 5, 7; 0.8)$, $C = (5, 7, 9, 10)$, $D = (6, 7, 9, 10; 0.6)$ and $E = (7, 8, 9, 10; 0.4)$.

To compare some of the other methods in [14], the readers can refer to Table 2.

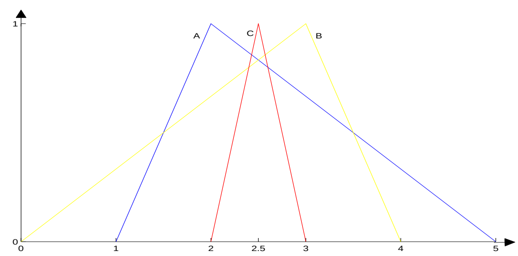

Fig. 4. Set 1.

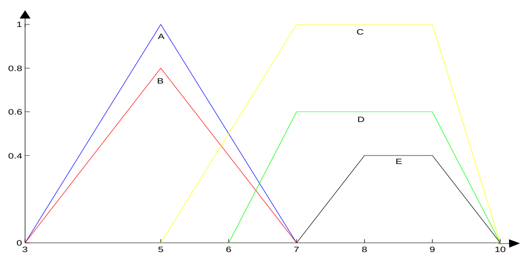

Fig. 5. Set 2.

Table 2. Comparative results of Example 4.2.

FNs	Proposed method	Sign distance p=2	Distance Minimization	Chu and Tsao (Revisited)	CV index	Magnitude method
Set 1.						
A	$D(A,B)=-2.5$	3.9157	2.5	0.74	0.32	2.16
B	$D(B,C)=-8$	3.9157	2.5	0.74	0.36	2.83
C	$D(A,C)=-10.5$	3.5590	2.5	0.75	0.08	2.5
Results	$A \prec B \prec C$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$	$B \prec A \prec C$	$A \prec C \prec B$
Set 2.						
A	$D(A,B)=7.5$	7.25	5.00	2.50	0.16	5.00
B	$D(B,C)=-25.5$	6.49	5.00	2.00	0.16	5.00
C	$D(C,D)=7.2$	11.54	7.75	3.89	0.14	7.91
D	$D(D,E)=3.12$	8.92	8.00	2.40	0.10	8.00
E	$D(A,E)=-13.8$	7.65	8.00	1.70	0.06	8.50
	$B \prec A \prec E$	$B \prec A \prec E$	$A \sim B \prec C$	$E \prec B \prec D$	$E \prec D \prec C$	$A \sim B \prec C$
	$E \prec D \prec C$	$E \prec D \prec C$	$C \prec D \sim E$	$D \prec A \prec C$	$C \prec A \sim B$	$C \prec D \prec E$

In Liou and Wang [19] ranking method, there was $A \sim B \prec C \prec D \prec E$. In Chu and Tsao [14] ranking method, there was $E \sim B \prec D \prec A \prec C$. In Ching-Hsue Cheng [11] ranking method, there was $B \prec A \prec C \prec D \prec E$. By using proposed method, there is $B \prec A \prec E \prec D \prec C$. From Fig. 5, the researchers can conclude that this order is more consistent with human intuition.

Example 4.3. Consider the following sets:
Set 1: $A_1 = (0.6, 0.7, 0.8)$, $A_2 = (0.4, 0.5, 0.6, 0.7)$, $A_3 = (0.2, 0.5, 0.8)$,
$A_4 = (0.3, 0.4, 0.9)$, $A_5 = (0.1, 0.2, 0.3)$.
Set 2: $B_1 = (0.1, 0.3, 0.5)$, $B_2 = (0.2, 0.3, 0.4)$.
Set 3: $C_1 = (0.2, 0.5, 0.8)$, $C_2 = (0.3, 0.4, 0.9)$.

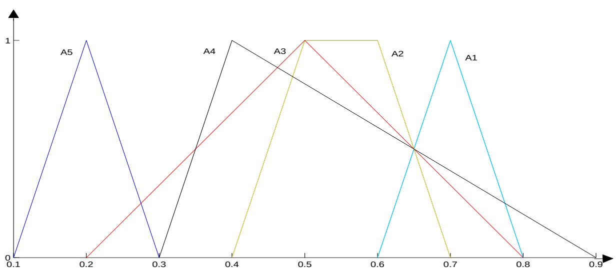

Fig. 6. Set 1.

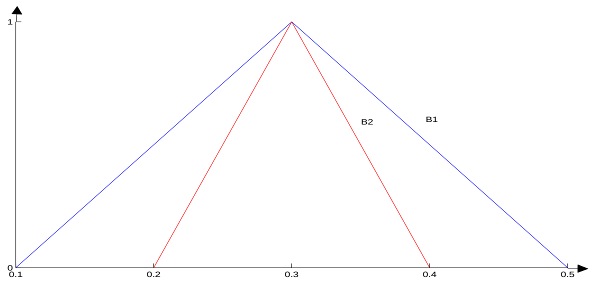

Fig. 7. Set 2.

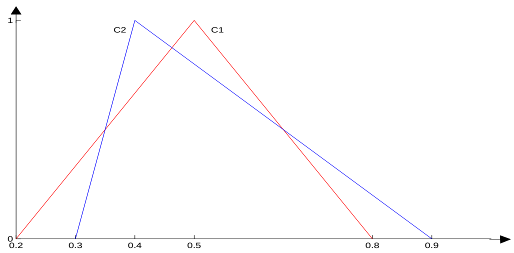

Fig. 8. Set 3.

In Chen and Lu [7] ranking method, there were $A_1 \succ A_2 \succ A_3 \sim A_4 \succ A_5$, when $\beta = 0.0$ and $A_1 \succ A_2 \succ A_3 \sim A_4 \succ A_5$, when $\beta = 0.5$ and $A_1 \succ A_2 \sim A_3 \sim A_4 \succ A_5$, when $\beta = 1.0$. By using proposed method, the ranking order is $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$. From Fig. 6, we can conclude that this order is more consistent with human intuition.

In Chu and Tsao [7], Asady and Zendehnam [3], Yager [28], Wang centroid method [26] and Baldwin and Guild [5] ranking methods, there were $B_1 \sim B_2$. In Chen [9] ranking method, there was $B_1 \prec B_2$. By using proposed method, there is $B_1 \prec B_2$. From Fig. 7, we can conclude that $B_1 \prec B_2$ is more consistent with human intuition.

Table 3. Comparative results of Example 4.3.

Authors	Set1	Set2	Set3
Proposed method	$D(A_1, A_3) = 4.07$	$D(B_1, B_2) = -0.45$	$D(C_1, C_2) = 0.3$
	$D(A_2, A_3) = 1.85$		
	$D(A_4, A_3) = -0.3$		
	$D(A_5, A_3) = -0.43$		
	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	$B_2 \succ B_1$	$C_1 \succ C_2$

Example 4.4. In this example, we investigate special types of fuzzy number and rank them.
Set 1: $H_1 = (1, 2, 2), H_2 = (2, 2, 3)$ (the fuzzy numbers are symmetric and positive, also their support have equal length).
Set 2: $K_1 = (-3, -1, 1, 3), K_2 = (-2, -1, 1, 2)$ (two symmetric trapezoidal fuzzy numbers such

that a part of their supports are negative).
Set 3: $L_1 = (-4, -3, -1, 0), L_2 = (-3, -2, -1)$ (the negative fuzzy number).

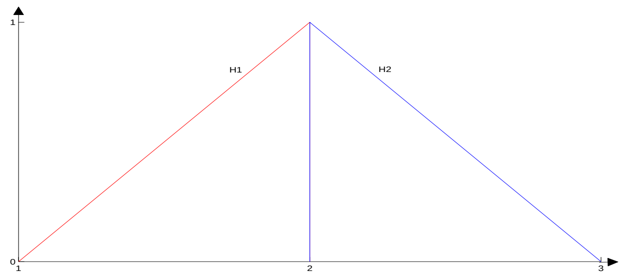

Fig. 9. Set 1.

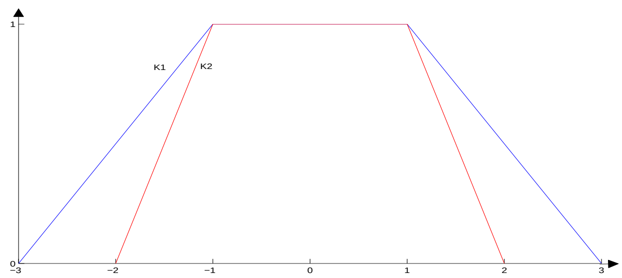

Fig. 10. Set 2.

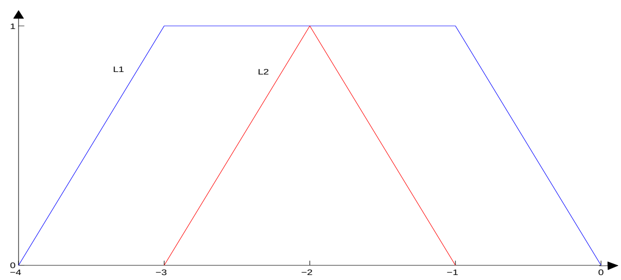

Fig. 11. Set 3.

It is clear that because of $D(H_1, H_2) = 0$ and $\overline{supp(H2)} > \overline{supp(H1)}$, we have $H2 \succ H1$. Also, for ranking set2 and set3, we first move all of the fuzzy numbers with values $|-3|$ and $|-4|$ to right side of x-axis, respectively, and then we rank all of them.

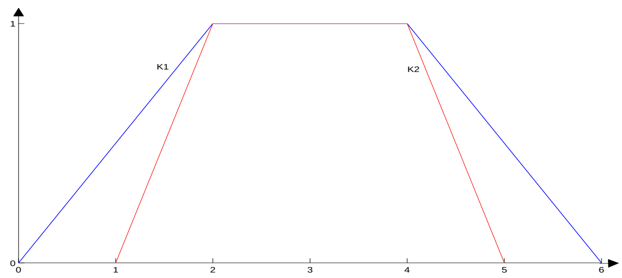

Fig. 12.

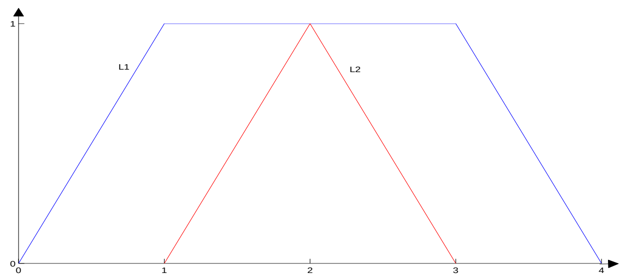

Fig. 13.

Table 4. Comparative results of Example 4.4.

Authors	Set1	Set2	Set3
Proposed method	$D(H_1, H_2) = 0$ $H_1 \prec H_2$	$D(K_1, K_2) = -2.75$ $K_1 \prec K_2$	$D(L_1, L_2) = -4$ $L_1 \prec L_2$

Example 4.5. Consider the triangular fuzzy number $A = (1, 2, 5)$, and the general number B, shown in Fig. 14., that the membership function of B is defined by

$$B(x) = \begin{cases} \sqrt{1-(x-2)^2} \; when \; x \in [1, 2], \\ \sqrt{1-\frac{1}{4}(x-2)^2} \; when \; x \in [2, 4], \\ 0 \; otherwise. \end{cases}$$

In Zhong-xing Wang and Ya-ni Mo [17] ranking method, there was $C^*_A = 0.4933$ and $C^*_B = 0.5067$, therefore $A \prec B$. By using proposed method, there is $D(A, B) = 2 - 4.1886 = -2.1886$. Thus, the ranking order is $A \prec B$. From Fig. 14, we can conclude that $A \prec B$ is more consistent with human intuition.

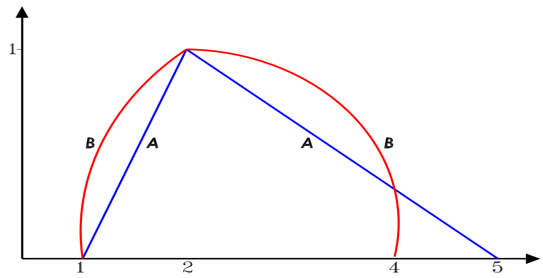

Fig. 14.

5. Some properties

In this section, some valuable properties which are useful in ranking a large quantity of fuzzy numbers simultaneously are described. Assume that there are m different bounded fuzzy numbers, $A_1, A_2, \dots A_m$, to be ranked. Let A_i, A_j, A_k be any three arbitrary fuzzy numbers, where $i \neq j \neq k$ and $1 \leq i, j, k \leq m$

(1) $D(A_i, A_i) = 0$.

(2) $D(A_i, A_j) = -D(A_j, A_i)$.

(3) If $D(A_i, A_j) > 0$ and $D(A_j, A_k) > 0$, then $D(A_i, A_k) > 0$.

Therefore, if $A_i > A_j$ and $A_j > A_k$ are known, we can infer that $A_i > A_k$. In fact, the total difference among three fuzzy numbers has the following relation: $D(A_i, A_k) = D(A_i, A_j) + D(A_j, A_k)$.

In other words, once the values of $D(A_i, A_j)$ and $D(A_j, A_k)$ are known, the value of $D(A_i, A_k)$ is determined by simple arithmetic computation.

(4) More than two fuzzy numbers can be ranked by comparing the benchmark fuzzy number. Let A_j be the benchmark, and $D(A_i, A_j) = a$ and $D(A_k, A_j) = b$. By using the previous two properties, obviously $D(A_i, A_k) = D(A_i, A_j) - D(A_k, A_j) = a - b$. Therefore, if $a > b$, then $D(A_i, A_k) > 0$; i.e. $A_i > A_k$.

(5) If $D(A_i, A_j) < \varepsilon$, then $|D(A_i, A_k) - D(A_j, A_k)| < \varepsilon$.

This equation suggests that the total difference between one fuzzy number and the other two fuzzy numbers are insignificant if the two fuzzy numbers are close to each other. This property holds, since $D(A_i, A_k) = D(A_i, A_j) + D(A_j, A_k)$. When using the above properties, for ranking m fuzzy numbers, only $m-1$ comparisons to the benchmark fuzzy number are necessary, instead of $m(m-1)/2$ or m comparisons. For example, letting A_j be the benchmark, then only $m-1$ values of the total difference, $D(A_1, A_j), D(A_2, A_j), ..., D(A_{j-1}, A_j), D(A_{j+1}, A_j), ..., D(A_m, A_j)$ are necessarily determined.

(6) This method can compare nonnormal fuzzy numbers. Thus the proposed approach is more efficient than the existing ranking methods in [7].

This article considers the following reasonable axioms that Wang and Kerre [23] proposed for ranking fuzzy quantities.

Let D be an ordering method, S the set of fuzzy quantities for which the method D can be applied, and $\mathcal{A}$ a finite subset of S. The statement "two elements A and B in $\mathcal{A}$ satisfy that A has a higher ranking than B when D is applied to the fuzzy quantities in $\mathcal{A}$" will be written as "$A \succ B$ by D on $\mathcal{A}$". "$A \sim B$ by D on $\mathcal{A}$", and "$A \succeq B$ by D on $\mathcal{A}$" are similarly interpreted. The axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach D are as follows [23]:

A-1. For an arbitrary finite subset $\mathcal{A}$ of S and $A \in \mathcal{A}$; $A \succeq A$ by D on $\mathcal{A}$.

A-2. For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B) \in \mathcal{A}^2$; $A \succeq B$ and $B \succeq A$ by D on $\mathcal{A}$, we should have $A \sim B$ by D on $\mathcal{A}$.

A-3. For an arbitrary finite subset $\mathcal{A}$ of S and $(A, B, C) \in \mathcal{A}^3$; $A \succeq B$ and $B \succeq C$ by D on $\mathcal{A}$, we should have $A \succeq C$ by D on $\mathcal{A}$.

A-4. Let S and S' be two arbitrary finite sets of fuzzy quantities in which D can be applied and A and B are in $S \cap S'$. We obtain the ranking order $A \succeq B$ by D on S' iff $A \succeq B$ by D on S.

A-5. Let A, B, $A + C$ and $B + C$ be elements of S. If $A \succeq B$ by D on A and B, then $A + C \succeq B + C$ by D on $A + C$ and $B + C$.

A$'$-5. Let A, B, $A + C$ and $B + C$ be elements of S. If $A \succ B$ by D on A and B, then $A + C \succ B + C$ by D on $A + C$ and $B + C$.

A-6. Let A, B, AC and BC be elements of S and $C \geq 0$. $A \succeq B$ by M on $\{A, B\}$ implies $AC \succeq BC$ by M on $\{AC, BC\}$.

The easy proofs of the properties mentioned above are left to the reader.

6. Conclusion

For ranking a large quantity of fuzzy numbers based on only limited information about them, an effective, efficient, and accurate ranking method becomes necessary. Most of methods which were proposed up to now have pitfalls in some aspects, such as inconsistency with human intuition and indiscrimination. In proposed method, the core and margin and $\alpha - cuts$ have been considered corresponding to each of fuzzy numbers simultaneously. Besides, all fuzzy numbers can be compared by our method, except real numbers. Also in this paper we described some properties which are useful in ranking a large quantity of fuzzy numbers. With respect to the examples, it is obvious that the proposed method is more reliable and computationally easier than some methods which were proposed up to now and overcome the shortcoming of the previous methods. Also, the proposed method can rank normal/nonnormal triangular and trapezoidal fuzzy numbers, but some methods, which were proposed up to now, were not able to do this.

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References

[1] Abbasbandy, S., Hajjari, T., (2009), A new approach for ranking of trapezoidal fuzzy numbers, Computers and Mathematics with Applications, 57(3), pp.413-419.

[2] Abbasbandy, S., Asady, B., (2006), Ranking of fuzzy numbers by sign distance, Information Sciences, 176(16), pp.2405-2416.

[3] Asady, A., Zendehnam, A., (2007), Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling, 31(11), pp.2589-2598.

[4] Asady, A., (2010), The revised method of ranking LR fuzzy number based on deviation degree, Expert Systems with Applications, 37(7), pp.5056-5060.

[5] Baldwin, J.F., Guild, N-C-F., (1979), Comparison of fuzzy numbers on the same decision space, Fuzzy Sets and Systems, 2, pp.213-233.

[6] Bortolan, G., Degani R., (1985), A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems, 15, pp.1-19.

[7] Chen, L-H., Lu, H-W., (2001), An approximate approach for ranking fuzzy numbers based on left and right dominance, Computers and Mathematics with Applications, 41(12), pp.1589-1602.

[8] Chen, S.J., Hwang, C.L., (1992), Fuzzy Multiple Attribute Decision Making: Methods and Applications, Springer-Verlang Berlin.

[9] Chen, S-H., (1985), Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17, pp.113-129.

[10] Cheng, C.H., (1998), A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95, pp.307-317.

[11] Cheng, C-H., (1999), Ranking alternatives with fuzzy weights using maximizing set and minimizing set, Fuzzy Sets and System, 105, pp.365-375.

[12] Choobineh, F., Li, H., (1993), An index for ordering fuzzy numbers, Fuzzy Sets and Systems, 54, pp.287-294.

[13] Chu, H., Lee-Kwang, H., (1994), Ranking fuzzy values ewith satisfaction function, Fuzzy Sets and Systems, 64, pp.295-311.

[14] Chu, T., Tsao, C., (2002), Ranking fuzzy numbers with an area between the centroid point and original point, Comput. Math. Appl., 43, pp.111-117.

[15] Deng, Y., Zhenfu, Z., Qi, L., (2006), Ranking fuzzy numbers with an area method using radius of gyration Computers and Mathematics with Applications, 51(6-7), pp.1127-1136.

[16] Garcia-Cascales, M.S., Lamata, M.T., (2007), A modification to the index of Liou and Wang for ranking fuzzy number, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, pp.411-424.

[17] Hong-xing Wang, Ya-ni Mo, (2010), Ranking Fuzzy Numbers Based on Ideal Solution. Fuzzy Inf. Eng., 1, pp.27-36.

[18] Kauffman, A., Gupta, M-M., (1991), Introduction to Fuzzy Arithmetic: Theory and Application, Van Nostrand Reinhold, New York.

[19] Liou, T-S., Wang, M.J., (1992), Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems, 50, pp.247-255.

[20] Liu, X-W., Han, S-L., (2005), Ranking fuzzy numbers with preference weighting function expectations Computers and Mathematics with Applications, 49(11-12), pp.1731-1753.

[21] Liu, Y-J., Fan, Z,P., Feng, B., (2009), Ranking L-R fuzzy number based on deviation degree, Information Sciences, 179(13), pp.2070-2077.

[22] Sun, H., Wu, J., (2006), A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method, Applied Mathematics and Computation, 174(1), pp.755-767.

[23] Wang, X., Kerre, E-E., (2001), Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Systems, 118(3), pp.375-385.

[24] Wang, X., Kerre, E-E., (2001), Reasonable properties for the ordering of fuzzy quantities (II), Fuzzy Sets and Systems 118, pp.387-405.

[25] Wang, Y, M., Luo, Y., (2009), Area ranking of fuzzy numbers based on positive and negative ideal points , Computers and Mathematics with Applications, 58(9), pp.1769-1779.

[26] Wang, Y. M., Yang, J.B., Xu D. L., (2006), On the centroids of fuzzy numbers. Fuzzy Sets and Systems 157, pp.919-926.

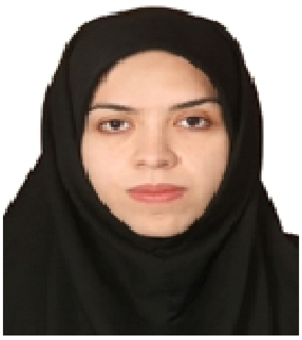
[27] Wang, Y-J., Lee, H-S., (2008), The revised method of ranking fuzzy numbers with an area between the centroid and original points, Computers and Mathematics with Applications, 55(9), pp.2033-2042.

[28] Yager R. R., (1981), A procedure for ordering fuzzy subsets of the unit interval. Information Science 24, pp.139-157.

[29] Yao, J., Wu, K., (2000), Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems, 116, pp.275-288.

Reza Ezzati was born in Tabriz, Iran in 1974. He received his B.Sc. degree in pure mathematics in 1997 and M.Sc. degree in applied mathematics from Azerbaijan Teacher Education University and Tarbyat Modarres University of Tehran, respectively, and his Ph.D. degree in applied mathematics from Islamic Azad University, Science and Research Branch, Tehran, Iran in 2006. He is an assistant professor in the Department of Mathematics at Islamic Azad University, Karaj Branch, (Iran). His current interests include fuzzy mathematics, especially, on numerical solution of fuzzy systems and fuzzy interpolation, iterative methods for solving nonlinear equations.

Ramin Enayati was born in Tehran, Iran, in 1982. He received the M.S. degree in applied mathematics from Department of Mathematics, Islamic Azad University, Science and Research Branch, Tehran, Iran in 2008. Since 2009, he is Ph.D. research student in Operation Research at Islamic Azad University, Karaj, Iran. His research interests include operation research, MOLP problems and fuzzy logic.



Azam Mottaghi was born in Iran in 1983. She received M.Sc. from Islamic Azad University, Mashhad Branch in 2008. She is currently doing her Ph.D research under supervision of Dr. Esmaeil Khorram and Dr. Reza Ezzati at the Department of Mathematics, Islamic Azad University, Karaj Branch. Her research interests include multi objective programming with fuzzy variable, nonlinear programming problem.

Rahim Saneifard was born in Oroumieh, Iran in 1972. He received B.Sc. in pure mathematics in 1997 and M.Sc. in applied mathematics from Azerbaijan Teacher Education University in Tabriz, Islamic Azad University and from Lahijan Branch in Lahijan, respectively. He is a Ph.D. student in the Department of Mathematics at Islamic Azad University, Science and Research Branch, Tehran, in Iran. His current interest include fuzzy mathematics.