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DYNAMICAL (TIME-HARMONIC) AXISYMMETRIC STRESS FIELD IN THE PRE-STRETCHED NON-LINEAR ELASTIC BI-LAYERED SLAB RESTING ON THE RIGID FOUNDATION

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Abstract. Within the framework of the piecewise homogeneous body model with the use of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies the axisymmetric dynamical stress field in the pre-stretched bi-layered slab resting on the rigid foundation is studied. It is assumed that the normal force which is located on a point and is time-harmonic acts on the free face plane of the slab. The elasticity relations of the layers' materials are described by the Murnaghan potential. The numerical results referring to the normal stress acting on the interface planes are presented for the pair of materials Aluminium 1915 and Acrylic Plastic.

Keywords: Bi-layered slab, initial strains, Hankel integral presentation, non-linear elastic, Murnaghan potential, time-harmonic stress field.

AMS Subject Classification: 17A32, 17A36.

1. Introduction

A class of interesting and urgent elastodynamic problems, for which the classical linear theory of elastic waves is not sufficient involves initially stressed bodies. Such problems have a wide range of application in practice. For example, initial stresses occur in structural elements after manufacturing and assembly. Initial stresses are also present in Earth's crust due to action of geostatic and geodynamic forces, in composites, in rocks and so on. Therefore, up to now, a large number of theoretical and experimental investigations had been made in this field. The systematic consideration and analyses of these results were made in [9]. The review of the recent researches is given in the papers [2, 5, 6, 10, 12]. It follows from these reviews that almost all these investigations were made within the framework of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB) and a considerable part of those refers to the wave propagation in the layered composite materials with homogeneous initial stresses.

The study of the influence of the initial stresses on the dynamic stress-state in a homogeneous and layered medium is of great significance, in both theoretical and practical sense. Until now there were few studies (for example, [1, 3, 4, 7, 8, 11, 13]). In the papers [11, 13] the Lamb's problem for a half-space with initial stresses was considered. In [3, 4] an attempt was made to study time-harmonic 2D Lamb's problem for the half-plane covered with the pre-stretched layer. In the paper [1] the investigations [3, 4] have been developed for a strip load on the covering layer. The development of the studies [3,4] for the 3D Lamb's problem has been made in [7].

In the foregoing investigations which regard the stress distribution in the pre-stressed body it is assumed that the region occupied by this body is semi-infinite. Therefore the results obtained in aforementioned investigations cannot be applied, for example, in the case where the dynamical stress field is studied for the layered material which rests on the rigid foundation. Nor can these

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results be applied for structural elements whose basic material is covered with layered ones. If the stiffness of the basic material (modulus of elasticity) is significantly greater than that of the covering layers, then the basic material can be modelled as a rigid foundation. As a result of the covering procedure the residual (initial) stresses arise in the covering layers and it is almost inevitable to avert these stresses. Therefore under studying the dynamical stress field in such structural members it is necessary to take the foregoing initial stresses into account.

Because of the above discussions in the present paper the investigations carried out in [1, 3, 4, 7, 8] are developed for the studying of the axisymmetric stress field in the pre-stretched bi-layered slab resting on the rigid foundation. It is assumed that the materials of the layers are non-linear elastic and the elasticity relations for these materials are given through the Murnaghan potential [14]. Also, it is assumed that the time-harmonic point-located load acts on the free face plane of the upper layer of the slab. The studies are made within the framework of the piecewise homogeneous bodies model by the use TLTEWISB.

2. Formulation of the problem

We consider the bi-layered slab resting on the rigid foundation and assume that in the natural state the thickness of the upper and lower layers of the slab are $h_1$and h_2 respectively. In the natural state we determine the position of the points of the layers by the Lagrangian coordinates in the Cartesian system of coordinates $Oy_1y_2y_3$ as well as in the cylindrical system of coordinates $Or\theta z$. We aim that the layers be stretched separately in the radial direction and each of them the homogeneous axisymmetric (with respect to Oy_3 or Oz axis) initial state appear.

With the initial state of the layers we associate the Lagrangian cylindrical system of coordinates $O'r'\theta' z'$ and Cartesian system of coordinates $O'y_1'y_2'y_3'$. The values related to the upper and lower layers are denoted by upper indices (1) and (2) respectively. Furthermore, the values related to the initial state are denoted by the upper index "0". Thus, according to the above-stated the initial state in the layers can be written as follows:

$$u_m^{(k),0} = \left(\lambda_m^{(k)} - 1\right) y_m,\ \lambda_m^{(k)} = const_m,\ \lambda_1^{(k)} = \lambda_2^{(k)}, m = 1,2,3,\ k = 1,2, \tag{1}$$

where $\lambda_m^{(k)}$ is the elongation along the Oy_m axis.

It follows from (1) that

$$y_i' = \lambda_i^{(k)} y_i,\ r' = \lambda_1^{(k)} r,\ h_1' = \lambda_3^{(1)} h_1,\ h_2' = \lambda_3^{(2)} h_2. \tag{2}$$

Below the values related to the system of coordinates associated with the initial state, i.e. with $Oy_1'y_2'y_3'$ are denoted by upper prime.

We investigate the axisymmetric stress field in the slab in the first variant of small initial strains of the TLTEWISB [9]. In the considered case the homogeneous initial stresses $\sigma_{rr}^{(k),0} = \sigma_{\theta\theta}^{(k),0} = \sigma^{(k),0}$ arise in the layers. The relations between $\sigma^{(k),0}$ and $\lambda_m^{(k)}$ are determined as follows.

$$\left(\lambda_1^{(k)}\right)^2 = \left(\lambda_2^{(k)}\right)^2 = 1 + \left(\frac{\lambda^{(k)}}{\mu^{(k)}} + 2\right) \frac{\sigma^{(k),0}}{\mu^{(k)}} \frac{1}{\left(3\lambda^{(k)}/\mu^{(k)} + 2\right)},$$

$$\left(\lambda_3^{(k)}\right)^2 = 1 - \frac{\lambda^{(k)}}{\mu^{(k)}} \frac{\sigma^{(k),0}}{\mu^{(k)}} \frac{1}{\left(3\lambda^{(k)}/\mu^{(k)} + 2\right)}, \tag{3}$$

where $\lambda^{(k)}$ and $\mu^{(k)}$ are Lam's constants of the materials. Thus, according to [9], we write the basic relations of the TLTEWISB for the axisymmetrical case.

The equation of motion.

$$\frac{\partial}{\partial r'} T'^{(k)}_{r'r'} + \frac{\partial}{\partial z'} T'^{(k)}_{r'z'} + \frac{1}{r'} \left(T'^{(k)}_{r'r'} - T'^{(k)}_{\theta'\theta'}\right) = \rho'^{(k)} \frac{\partial^2}{\partial t^2} u'^{(k)}_{r'},$$

$$\frac{\partial}{\partial r'}T'^{(k)}_{z'r'}+\frac{1}{r'}T'^{(k)}_{z'r'}+\frac{\partial}{\partial z'}T'^{(k)}_{z'z'}=\rho'^{(k)}\frac{\partial^2}{\partial t^2}u'^{(k)}_{z'}. \quad (4)$$

The mechanical relations

$$T'^{(k)}_{r'r'}=\omega'^{(k)}_{1111}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\omega'^{(k)}_{1122}\frac{u'^{(k)}_{r'}}{r'}+\omega'^{(k)}_{1133}\frac{\partial u'^{(k)}_{z'}}{\partial z'},$$

$$T'^{(k)}_{\theta'\theta'}=\omega'^{(k)}_{2211}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\omega'^{(k)}_{2222}\frac{u'^{(k)}_{r'}}{r'}+\omega'^{(k)}_{2233}\frac{\partial u'^{(k)}_{z'}}{\partial z'},$$

$$T'^{(k)}_{r'z'}=\omega'^{(k)}_{1313}\frac{\partial u'^{(k)}_{r'}}{\partial z'}+\omega'^{(k)}_{1331}\frac{\partial u'^{(k)}_{z'}}{\partial r'},T'^{(k)}_{z'r'}=\omega'^{(k)}_{3113}\frac{\partial u'^{(k)}_{r'}}{\partial z'}+\omega'^{(k)}_{3131}\frac{\partial u'^{(k)}_{z'}}{\partial r'},$$

$$T'^{(k)}_{z'z'}=\omega'^{(k)}_{3311}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\omega'^{(k)}_{3322}\frac{u'^{(k)}_{r'}}{r'}+\omega'^{(k)}_{3333}\frac{\partial u'^{(k)}_{z'}}{\partial z'}. \quad (5)$$

In (4) and (5) through $T'^{(k)}_{r'r'}$, ..., $T'^{(k)}_{z'z'}$ the perturbations of the components of Kirchoff stress tensor are denoted, notation $u'^{(k)}_{r'}$, $u'^{(k)}_{z'}$ shows the perturbation of the components of the displacement vector. The constants $\omega'^{(k)}_{1111}$, ..., $\omega'^{(k)}_{3333}$, $\rho'^{(k)}$ in (4), (5) are determined through the mechanical constants of the layers' materials and through the initial stress state. The expressions for these constants will be given below.

In the present investigation we assume that the mechanical relations of the layers' materials are described by the Murnaghan potential [14], which is given as follows.

$$\Phi^{(k)}=\frac{1}{2}\lambda^{(k)}\left(A_1^{(k)}\right)^2+\mu^{(k)}A_2^{(k)}+\frac{a^{(k)}}{3}\left(A_1^{(k)}\right)^2+b^{(k)}A_1^{(k)}A_2^{(k)}+\frac{c^{(k)}}{3}A_3^{(k)}. \quad (6)$$

In (6) $\lambda^{(k)}$ and $\mu^{(k)}$ are Lam's, $a^{(k)}$, $b^{(k)}$ and $c^{(k)}$ are third order elastic constants; $A_1^{(k)}$, $A_2^{(k)}$ and $A_3^{(k)}$ are the 1-st, 2-nd and 3-rd algebraic invariants of Green's strain tensor respectively.

According to [9], for the considered case the expressions for $\omega'^{(k)}_{1111}$, ..., $\omega'^{(k)}_{3333}$, $\rho'^{(k)}$ can be written as follows:

$$\omega'^{(k)}_{1111}=\omega'^{(k)}_{2222}=\omega'^{(k)}_{2211}=\omega'^{(k)}_{1122}=\left(\lambda_3^{(k)}\right)^{-1}\left(\left(\lambda_1^{(k)}\right)^2A_{11}^{(k)}+\sigma^{(k),0}\right),$$

$$\omega'^{(k)}_{3333}=\left(\lambda_3^{(k)}\right)^2\left(\lambda_1^{(k)}\right)^{-2}A_{33}^{(k)},$$

$$\omega'^{(k)}_{3311}=\omega'^{(k)}_{1133}=\omega'^{(k)}_{2233}=\lambda_3^{(k)}A_{13}^{(k)},\omega'^{(k)}_{1331}=\left(\lambda_3^{(k)}\right)^{-1}\left(\left(\lambda_3^{(k)}\right)^2\mu_{13}^{(k)}+\sigma^{(k),0}\right),$$

$$\omega'^{(k)}_{1313}=\omega'^{(k)}_{3131}=\lambda_3^{(k)}\mu_{13}^{(k)},\quad \omega'^{(k)}_{3113}=\lambda_3^{(k)}\mu_{13}^{(k)}, \quad (7)$$

where

$$\sigma^{(k),0}=\sigma_{rr}^{(k),0}=\sigma_{\theta\theta}^{(k),0},A_{11}^{(k)}=\left(\lambda^{(k)}+2\mu^{(k)}\right)\left(1+\frac{\sigma^{(k),0}}{\left(\lambda^{(k)}+2\mu^{(k)}\right)3K_0^{(k)}}\left(4a^{(k)}+\frac{\lambda^{(k)}+4\mu^{(k)}}{\mu^{(k)}}2b^{(k)}+\right.\right.$$

$$\left.\left.+\frac{\lambda^{(k)}+2\mu^{(k)}}{\mu^{(k)}}c^{(k)}\right)\right),A_{33}^{(k)}=\left(\lambda^{(k)}+2\mu^{(k)}\right)\left(1+\frac{\sigma^{(k),0}}{\left(\lambda^{(k)}+2\mu^{(k)}\right)3K_0^{(k)}}\left(4a^{(k)}-\frac{\lambda^{(k)}-\mu^{(k)}}{\mu^{(k)}}4b^{(k)}+\right.\right.$$

$$\left.\left.+\frac{\lambda^{(k)}}{\mu^{(k)}}2c^{(k)}\right)\right),A_{13}^{(k)}=\lambda^{(k)}\left(1+\frac{\sigma^{(k),0}}{3K_0^{(k)}\lambda^{(k)}}(4a^{(k)}-\frac{\lambda^{(k)}-2\mu^{(k)}}{\mu^{(k)}}b^{(k)}\right)\right),\mu_{13}^{(k)}=\mu\left(1+\frac{\sigma^{(k),0}}{3K_0^{(k)}\mu^{(k)}}\times\right.$$

$$\times\left(2b^{(k)}-\frac{\lambda^{(k)}-2\mu^{(k)}}{\mu^{(k)}}\frac{c^{(k)}}{4}\right)\Bigg)\,,K_0^{(k)}=\lambda^{(k)}+\frac{2}{3}\mu^{(k)}. \tag{8}$$

Thus the considered dynamical stress distribution will be investigated by the use of the equations (3)-(8). In this case we will assume that the following contact and boundary conditions are satisfied.

$$T'^{(k)}_{z'z'}\Big|_{z'=0}=-P_0\delta(r')e^{i\omega t},\,T'^{(k)}_{z'r'}\Big|_{z'=0}=0,\,T'^{(1)}_{z'z'}\Big|_{z'=-h'_1}=T'^{(2)}_{z'z'}\Big|_{z'=-h'_1},$$

$$T'^{(1)}_{z'r'}\Big|_{z'=-h'_1}=T'^{(2)}_{z'r'}\Big|_{z'=-h'_1},,$$

$$u'^{(1)}_{r'}\Big|_{z'=-h'_1}=u'^{(2)}{}_{r'}\Big|_{z'=-h'_1},\,u'^{(1)}_{z'}\Big|_{z'=-h'_1}=u\,'^{(2)}_{z'}\Big|_{z'=-h'_1},\,u'^{(2)}_{r'}\Big|_{z'=-h'_2}=0,\,u'^{(2)}_{z'}\Big|_{z'=-h'_2}=0. \tag{9}$$

3. Method of solution

For the solution to the considered problem the following representation for the displacements is used [9]:

$$u'^{(k)}_{r'}=-\frac{\partial^2}{\partial r'\partial z'}\mathrm{X}^{(k)},u'^{(k)}_{z'}=\frac{1}{\omega'^{(k)}_{1133}+\omega'^{(k)}_{1313}}\left(\omega'^{(k)}_{1111}\Delta'_1+\omega'^{(k)}_{3113}\frac{\partial^2}{\partial z'^2}-\rho'^{(k)}\frac{\partial^2}{\partial t^2}\right)\mathrm{X}^{(k)}\,, \tag{10}$$

where $\mathrm{X}^{(k)}$ satisfies the equation

$$\Bigg[\left(\Delta'+\left(\xi'_2\right)^2\frac{\partial^2}{\partial z'^2}\right)\left(\Delta'+\left(\xi'_3\right)^2\frac{\partial^2}{\partial z'^2}\right)-\rho'^{(k)}\times$$
$$\times\left(\frac{\omega'^{(k)}_{1111}+\omega'^{(k)}_{1331}}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}}\Delta'+\frac{\omega'^{(k)}_{3333}+\omega'^{(k)}_{3113}}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}}\frac{\partial^2}{\partial z'^2}\right)\frac{\partial^2}{\partial t^2}+$$
$$+\frac{\left(\rho'^{(k)}\right)^2}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}}\frac{\partial^4}{\partial t^4}\Bigg]\mathrm{X}^{(k)}=0. \tag{11}$$

In (10), (11) the following notation is used:

$$\Delta'=\frac{d^2}{dr'^2}+\frac{1}{r'}\frac{d}{dr'},\left(\xi'^{(k)}_{2,3}\right)^2=d^{(k)}\pm\left[\left(d^{(k)}\right)^2-\omega'^{(k)}_{3333}\omega'^{(k)}_{3113}\left(\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}\right)^{-1}\right]^{\frac{1}{2}},$$

$$d^{(k)}=\left(2\omega'^{(k)}_{1111}\omega'^{(k)}_{1311}\right)^{-1}\left[\omega'^{(k)}_{1111}\omega'^{(k)}_{3333}+\omega'^{(k)}_{1331}\omega'^{(k)}_{3113}-\left(\omega'^{(k)}_{1133}+\omega'^{(k)}_{1313}\right)^2\right]. \tag{12}$$

Now we consider the solution to equation (11). Because the point load is harmonic in time, only the stationary case will be considered; all dependent variables become harmonic and can be represented as:

$$\left\{T'^{(k)}_{r'r'},...,T'^{(k)}_{z'z'},u'^{(k)}_{r'},u'^{(k)}_{z'},\mathrm{X}^{(k)}\right\}=\left\{\overline{T}'^{(k)}_{r'r'},...,\overline{T}'^{(k)}_{z'z'},\overline{u}'^{(k)}_{r'},\overline{u}'^{(k)}_{z'},\overline{\mathrm{X}}^{(k)}\right\}e^{i\omega t}, \tag{13}$$

where a superimposed dash denotes the amplitude of the relevant quantity. From here on we will omit this superimposed dash. Thus, introducing the dimensionless coordinates $r'\to r'/h_1$, $z'\to z'/h_1$we obtain the following equation from (11), (13).

$$\Bigg[\left(\Delta'+\left(\xi'_2\right)^2\frac{\partial^2}{\partial z'^2}\right)\left(\Delta'+\left(\xi'_3\right)^2\frac{\partial^2}{\partial z'^2}\right)+\rho'^{(k)}\times$$

$$\times\left(\frac{\omega_{1111}^{\prime(k)}+\omega_{1331}^{\prime(k)}}{\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}}\Delta'+\frac{\omega_{3333}^{\prime(k)}+\omega_{3113}^{\prime(k)}}{\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}}\frac{\partial^2}{\partial z'^2}\right)\frac{\Omega^2\mu}{\rho'}-$$

$$-\left.\frac{\left(\rho'^{(k)}\right)^2\Omega^4(\mu)^2}{\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}(\rho')^2}\right]\mathrm{X}^{(k)}=0, \tag{14}$$

where

$$\Omega^2=\frac{(\omega h_1)^2\rho'}{\mu}, \mu=\min\left(\mu^{(1)},\mu^{(2)}\right), \rho'=\rho'^{(1)} if \mu=\mu^{(1)}, \rho'=\rho'^{(2)} if \mu=\mu^{(2)}. \tag{15}$$

For, the solution to equation (14) we use the Hankel integral presentation for the function $\mathrm{X}^{(k)}$:

$$\mathrm{X}^{(k)}=\int_0^\infty F^{(k)}(s)e^{\gamma^{(k)}z'}J_0(sr')sds, \tag{16}$$

where $J_0(x)$ is the Bessel function of zeroth order.

Substituting (16) into (14) and doing some mathematical manipulation we obtain that for satisfying equation (14) $\gamma^{(k)}$ must satisfy the algebraic equation

$$A^{(k)}\left(\gamma^{(k)}\right)^4+B^{(k)}\left(\gamma^{(k)}\right)^2+C^{(k)}=0, \tag{17}$$

where

$$A^{(k)}=\left(\xi_2^{\prime(k)}\right)^2\left(\xi_3^{\prime(k)}\right)^2, B^{(k)}=-s^2\left(\left(\xi_2^{\prime(k)}\right)^2+\left(\xi_3^{\prime(k)}\right)^2\right)+$$

$$+\frac{\Omega^2\mu\rho'^{(k)}}{\rho'\mu^{(k)}\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}}\left(\omega_{3333}^{\prime(k)}+\omega_{3113}^{\prime(k)}\right),$$

$$C^{(k)}=s^4-s^2\frac{\left(\omega_{1111}^{\prime(k)}+\omega_{1331}^{\prime(k)}\right)}{\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}}\frac{\Omega^2\mu\rho'^{(k)}}{\rho'}+\frac{\Omega^4\left(\mu\rho'^{(k)}\right)^2}{(\rho')^2\omega_{1111}^{\prime(k)}\omega_{1331}^{\prime(k)}}. \tag{18}$$

Denoting the roots of equations (17) through $\pm\gamma_1^{(k)}$, $\pm\gamma_2^{(k)}$ we attain the following expressions for the function $\mathrm{X}^{(k)}$:

$$X^{(k)}=\int_0^\infty\left[F_1^{(k)}(s)e^{\gamma_1^{(k)}z'}+F_2^{(k)}(s)e^{-\gamma_1^{(k)}z'}+F_3^{(k)}(s)e^{\gamma_2^{(k)}z'}+F_4^{(k)}(s)e^{-\gamma_2^{(k)}z'}\right]J_0(sr)sds. \tag{19}$$

From the conditions (9) we obtain the corresponding equations for determine the unknowns $F_1^{(k)}(s),...,F_4^{(k)}(s)$ which enter (19). Thus, by employing the algorithm developed in [1, 3, 4, 7, 8], we can calculate the values of the stresses and displacements.

4. Numerical results and discussions

The following materials are selected for numerical consideration: Acrylic Plastic (shortly AP) with properties $\rho=1.16g/cm^3$, $\lambda=0.4\times10^4MPa$, $\mu=0.19\times10^4MPa$, $a=-0.0391\times10^5MPa$, $b=-0.072\times10^5MPa$, $c=-0.141\times10^5MPa$; Aluminium 1915 (shortly Al 1915) with properties $\rho=2.77g/cm^3$, , $\mu=2.8\times10^4MPa$, $a=0.62\times10^5MPa$, $b=-0.49\times10^5MPa$, $c=-3.43\times10^5MPa$. These values of the mechanical constants which enter the expression of the Murnaghan potential (6) are taken from [9].

For testing the validity of the algorithm and programmes we consider the case where the slab consists of the single layer. Analyze the distribution of the stress $\sigma'_{z'z'}$ on the plane between the rigid foundation and slab. We examine the influence of the Ω (15) on this distribution. According to the mechanical consideration, under the absence of the initial stretching of the slab the values of $\sigma'_{z'z'}$ must approach the values obtained for the corresponding static problem studied in [15] as $\Omega \to 0$.

Now we consider the comparison of the present results with the corresponding ones obtained by the use of the integral expression given for σ_{zz} in [15] and assume that the Poisson ratio ν of the material of the single-layer-slab is equal to 0.3 which corresponds to Al. Fig. 1 shows the graphs of the dependencies between $\sigma'_{z'z'}h_1^2/P_0$ and r'/h_1 (h_1 being a thickness of the slab) for various Ω. It follows from these graphs that the values of $\sigma'_{z'z'}h_1^2/P_0$ obtained for the dynamical problem approach the corresponding ones obtained for the static problem as $\Omega \to 0$. This situation holds for the correctness of the used algorithm and programmes.

We consider the influence of the initial pre-stretching of the single-layer slab on the dependencies between $\sigma'_{z'z'}h_1^2/P_0$ (at $r'/h_1 = 0$) and Ω. The graphs of these dependencies are given in Fig. 2a (for Al 1915) and in Fig. 2b (for AP). It follows from these graphs that for the considered range of the change of Ω, i.e. for $0 < \Omega \leq 1.6$ the absolute values of $\sigma'_{z'z'}h_1^2/P_0$ increase monotonically with Ω. In this case the pre-stretching of the slab causes a decrease in these values. Moreover it follows from these results that as a result of the influence of the third order elastic constants the absolute values of the $\sigma'_{z'z'}h_1^2/P_0$ also decrease.

Now we analyze the numerical results regarding the bi-layered slab and consider the following two cases: the case I AP (upper layer) + Al 1915 (lower layer); the case II al 1915 (upper layer) + AP (lower layer). Assume that $h_1 = h_2$. The graphs of the dependencies between $\sigma'_{z'z'}h_1^2/P_0$ (at $z' = -h'_1$, $r'/h_1 = 0$) and Ω are given in Fig. 3a and 3b for the cases I and II respectively. Note that under construction of these graphs it is assumed that the initial stretching exists only in the upper layer of the slab. Because the various numerical investigations which are not given here show that the influence of the pre-stretching of the lower layer the values of the considered stress is insignificant.

Thus, it follows from the results given in Fig. 3a and Fig. 3b that the pre-stretching of the upper layer causes a decrease in the absolute values of the normal stress acting on the interface plane between the layers. In this case as a result of the non-linearity of the upper layer material the absolute values of the considered stress decrease (increase) in the case I (in the case II).

In the foregoing investigations the values of the Ω are bounded by 1.4 and 1.0 for cases I and II respectively. Because under $\Omega > 1.4$ ($\Omega > 1.0$) in case I (in case II) the resonance type behaviour of the considered slab is observed. The studying of such type behaviour of the considered slab will be subject to other investigations of the author.

Consider the distribution of the stress $\sigma'_{z'z'}h_1^2/P_0$ in the interface planes under $\Omega = 1.0$. The graphs of these distributions for case I (for case II) are given in Fig. 4a (in Fig. 4b). In these figures the graphs denoted by numbers 1, 2 and 3 ($1'$, $2'$ and $3'$) show the distribution of $\sigma'_{z'z'}h_1^2/P_0$ with respect to r'/h_1 at $z' = -h'_1$ (at $z' = -h'_1 - h'_2$) for $\sigma^{(1),0}/\mu^{(1)} = 0.008, 0.02$ and 0.04 respectively. It follows from these results that the main effect of the influence of the pre-stretching of the upper layer on the considered distribution arises in the near vicinity of the point $r'/h_1 = 0$ (i.e. in $r'/h_1 \leq 0.4$).

5. Conclusions

In the light of the discussed results the following conclusions can be drawn.

In the framework of the piecewise homogeneous body model with the use of the TLTEWISB the axisymmetrical dynamical time-harmonic stress field in the pre-stretched bi-layered slab resting on the rigid foundation is investigated. The elasticity relations of the layers' materials are described by the Murnaghan potential. Concrete numerical results are made for the pair of materials Aluminum 1915 and Acrylic Plastic. As a result of the numerical investigations the following conclusions are established:

-the absolute values of the interface normal stress increase monotonically with frequency of the external point-located normal force;

- the initial tension of the layers in the slab causes the absolute values of the interface normal stress to decrease;

- the influence of the non-linearity of the layers' materials depends on the location sequence of the layers in the slab: for the location Aluminium 1915 (upper layer) + Acrylic Plastic (lower layer) (Acrylic Plastic (upper layer) + Aluminium 1915 (lower layer)) this non-linearity causes the absolute values of the stress to decrease (to increase). But for the single-layer slab the nonlinearity of the considered materials causes the absolute values of the stress to decrease.

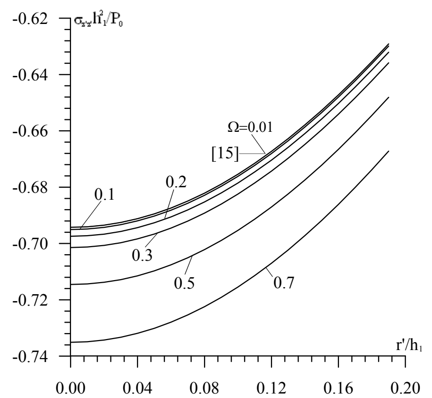
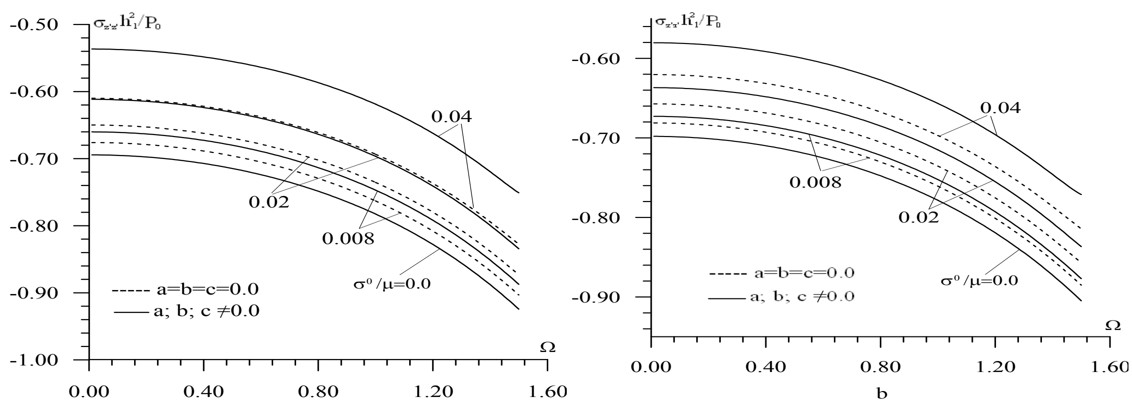
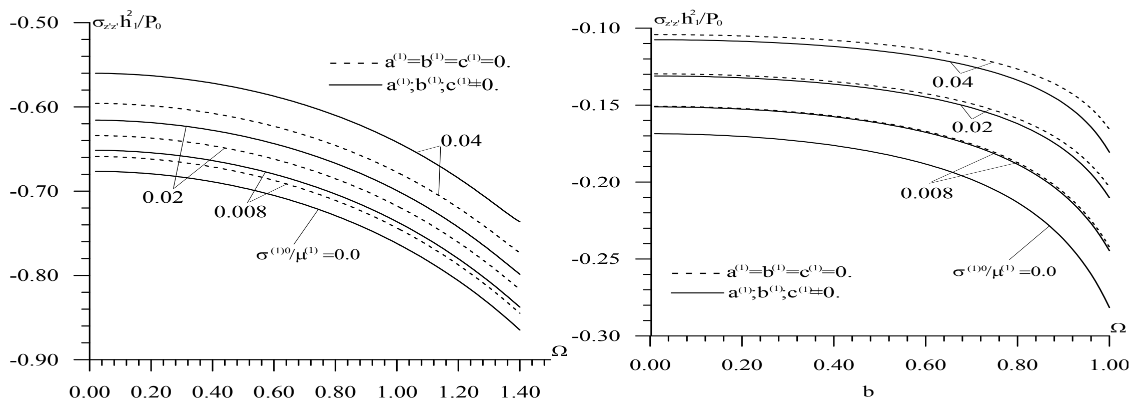


Fig. 1.



a b

Fig. 2.



a b

Fig. 3.

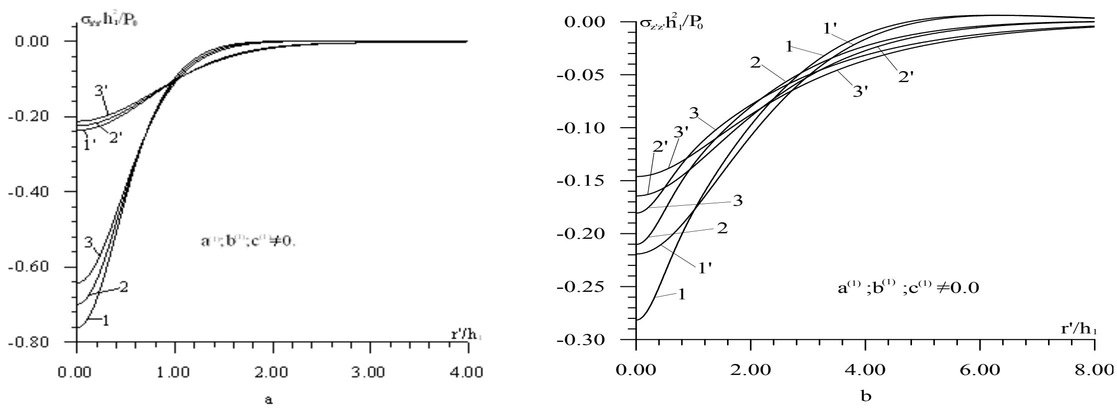


Fig. 4.

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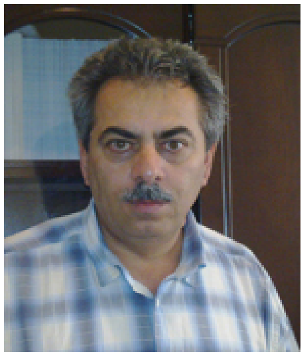

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