ON CLOSED MAPPINGS OF UNIFORM SPACES

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Abstract. In the paper the $u$−continuous, $u$−closed, $z_u$−closed, $u$−perfect mappings have been determined and their some properties have been established. The importance of these mappings classes is caused by that $u$−closed mappings are a subclass of the closed mappings class, and the closed mappings class is a subclass of the $z_u$−closed mappings.

Keywords: open (closed) sets, zero− (cozero)sets, uniformly continuous mapping, perfect mapping, bicompact.

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1. Introduction

Z. Frolik ([4]) introduced $z$-closed mappings, which are a natural generalization of the closed mappings ([3]).

Definition 1.1. ([4]). A continuous mapping $f : X \to Y$ of a topological space $X$ into a topological space $Y$ is called $z$−closed, if the image $f(F)$ of any functionally closed ($\equiv$ zero-set) $F$ in $X$ is closed set in $Y$.

Below the uniform analogues of closed and $z$− closed mappings have been determined. Everywhere necessary information and denotations are taken from books [6] and [1],[2],[5].

Every uniform space be $uX$, where $U$ be a uniformity in a uniform coverings terms, $f : uX \to vY$ be a mapping of uniform space $uX$ into uniform space $vY$ and if $f(F)$ = $Y$, then the mapping $f$ is surjective. We denote $C^*(uX)$ to be a ring of all bounded uniformly continuous functions on $uX$, $\mathcal{Z}(uX)$ = $\{f^{-1}(0) : f \in C^*(uX)\}$ be a set of all uniformly zero-sets ($\equiv$ uniformly closed sets ([2])), $\mathcal{L}(uX)$ = $\{f^{-1}(\mathbb{R}\{0\}) : f \in C^*(uX)\}$ be a set of all uniformly cozero-sets ($\equiv$ uniformly open sets ([2])) of the uniform space $uX$.

Let $u\mathbb{R}\mathbb{R}$ be a set of real numbers $\mathbb{R}$ with natural uniformity $U_{\mathbb{R}}$, generated by the metrics $\rho(x,y) = |x - y|$ for any $x, y \in \mathbb{R}$, and $uI$ be a segment $I = [0,1]$ with uniformity $U_I$, induced by the uniformity $U_{\mathbb{R}}$.

Definition 1.2. ([2]). A mapping $f : uX \to vY$ is called $u$−continuous, if the inverse image $f^{-1}(F) \in \mathcal{Z}(uX)(f^{-1}(U) \in \mathcal{L}(uX))$ for any $F \in \mathcal{Z}(uY)(U \in \mathcal{L}(uY))$.

Remark 1.1. Every uniformly continuous mapping $f : uX \to vY$ is $u$−continuous. If $U_f$ and $V_f$ are fine uniformities of Tychonoff spaces $X$ and $Y$ respectively, then for mapping $f : u_fX \to v_fY$ $u_f$−continuity is equivalent to the continuity of mapping $f : X \to Y$: There is $u$−continuous mapping $f : uX \to vY$, which is not uniformly continuous.

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Theorem 1.1. ([2]). Let \( g_1^{-1}(0) = F_1 \in \mathfrak{F}(uX) \) and \( g_2^{-1}(0) = F_2 \in \mathfrak{F}(uX) \), where \( g_1, g_2 \in C^*(uX) \) and \( F_1 \cap F_2 = \emptyset \). Then the function \( f : uX \to u_1I \), determined as \( f(x) = \lfloor g_1(x) \rfloor / \lceil |g_1(x)| + |g_2(x)| \rceil \) for any \( x \in X \), is a \( u \)-function.

Example 1.1. Let \( X = [-1; 0) \cup (0; 1] \) and a uniformity \( \mathcal{U} \) on \( X \) is induced by the uniformity \( \mathcal{U}_\mathbb{R} \) of \( \mathbb{R} \). The sets \([-1; 0) \) and \((0; 1]\) are no uniformly separated, hence, there is no uniformly continuous function on the uniform space \( uX \), which separates these sets. Functions \( g_i : uX \to u_\mathbb{R} \mathbb{R} \), \( i = 1, 2 \), determined as \( g_1(x) = \rho(x, (-1; 0)) \) and \( g_2(x) = \rho(x, (0; 1]) \) are uniformly continuous. Then the function \( f(x) = g_1(x) / (g_1(x) + g_2(x)) \) is an example of the \( u \)-continuous function, which is not uniformly continuous.

2. Main results

Example 2.1. Let \( \varepsilon > 0 \) and \( \mathbb{R}^+ = (0; +\infty) \). A uniformity \( \mathcal{U}_\mathbb{R} \) of real numbers \( \mathbb{R} \) is generated by the basis \( \mathbb{B} \), consisting of uniform coverings \( \alpha_\varepsilon = \{O_\varepsilon(x) : x \in \mathbb{R}\} \), where \( O_\varepsilon(x) = (x - \varepsilon, x + \varepsilon) \) is open interval with center at point \( x \) of length \( 2\varepsilon \). Let \( \mathcal{P}(\mathbb{R}) \) be a set of all finite subsets of \( \mathbb{R} \) and \( \mathbb{R}^+ \). For any \( \varepsilon \in \mathbb{R}^+ \) and any \( A \in \mathcal{P}(\mathbb{R}) \) suppose \( \alpha_{\varepsilon,A} = \{O_\varepsilon(x) \setminus A : x \in \mathbb{R} \setminus A\} \cup \{a : a \in A\} \). A family \( \mathbb{B}' = \alpha_{\varepsilon,A} : \varepsilon \in \mathbb{R}^+ \), \( A \in \mathcal{P}(\mathbb{R}) \) is a basis of some uniformity \( \mathcal{U}' \) on \( \mathbb{R} \), more strong, than uniformity \( \mathcal{U}_\mathbb{R} \). Really, \( \alpha_{\varepsilon, A_1} \land \alpha_{\varepsilon, A_2} > \alpha_{\varepsilon, A_1 \cup A_2} \), where \( \varepsilon = \min\{\varepsilon_1, \varepsilon_2\} \) and the covering \( \alpha_{\varepsilon, A} \) is starry inscribed to the covering \( \alpha_{\varepsilon, A} \), where \( \delta = \varepsilon / 3 \). We note, that \( \mathcal{U}_\mathbb{R} \subset \mathcal{U}' \) and \( \mathcal{U}' \) generates discrete topology on the \( \mathbb{R} \).

Proposition 2.1. A set of rational numbers \( \mathbb{Q} \) is not uniformly zero-set in the uniform space \( u\mathbb{R} \), i.e. \( \mathbb{Q} \notin \mathfrak{F}(u\mathbb{R}) \).

Proof. We suppose, that \( \mathbb{Q} \in \mathfrak{F}(u\mathbb{R}) \), i.e. there is such uniformly continuous function \( f \in C^*(u\mathbb{R}) \), that \( \mathbb{Q} = f^{-1}(0) \). Then for any \( n \in \mathbb{N} \) there exist \( \varepsilon_n > 0 \) and \( A_n \in \mathcal{P}(\mathbb{R}) \) such that the family \( f(\alpha_{\varepsilon_n, A_n}) \) is inscribed to the covering \( \alpha_{\varepsilon_n, A_n} \), i.e. for any \( y \in O_{\varepsilon_n}(x) \) the formula \( |f(x) - f(y)| < \frac{1}{n} \) is provided for all \( x \in \mathbb{R} \). Let \( x \notin A = \bigcup_{n=1}^\infty A_n \), in force of everywhere density of \( \mathbb{Q} \) in \( \mathbb{R} \), there is such \( y \in \mathbb{Q} \setminus A \), that \( |x - y| < \varepsilon_n \), hence for all \( x \in \mathbb{R} \) for all such \( x \notin A \) and \( |x - y| < \varepsilon_n \) we have \( |f(x)| < \frac{1}{n} \) for any \( n \in \mathbb{N} \), i.e. \( f(x) = 0 \) for any \( x \in \mathbb{R} \setminus A \). Thus, \( \mathbb{R} \setminus A = f^{-1}(0) \), i.e. \( \mathbb{R} = \mathbb{Q} \cup A \) is contradiction, since \( \mathbb{Q} \) and \( A \) are countable sets, and \( \mathbb{R} \) is uncountable. The proposition is proved.

We consider the function \( h : u\mathbb{R} \to u_\mathbb{R} \mathbb{R} \), determined as \( g(x) = 0 \), if \( x \in \mathbb{Q} \) and \( g(x) = 1 \), if \( x \in \mathbb{R} \setminus \mathbb{Q} \). Then \( h \) is continuous function, which is not \( u' \)-continuous function, since \( g^{-1}(0) = \mathbb{Q} \notin \mathfrak{F}(u\mathbb{R}) \). By means of example 2.1, it is naturally to determine a special closed mappings of uniform spaces.

Definition 2.1. A mapping \( f : uX \to uY \) is called \( u \)-closed if it is \( u \)-continuous and for any closed set \( F \) in \( X \) the image \( f(F) \) is closed in \( Y \).

Definition 2.2. A mapping \( f : uX \to uY \) is called \( z_u \)-closed, if it is \( u \)-continuous and for any uniformly closed set \( F \in \mathfrak{F}(uX) \) the image \( f(F) \) is closed in \( Y \).

Obviously, every \( u \)-closed mapping is \( z_u \)-closed. It takes place the next simple

Proposition 2.2. Every \( u \)-closed mapping is \( z_u \)-closed.

Theorem 2.1. A mapping \( f : uX \to uY \) is \( z_u \)-closed if and only if for every point \( y \in Y \) and every uniformly cozero-set \( U \in \mathcal{L}(uX) \), containing \( f^{-1}(y) \), i.e. \( f^{-1}(y) \subset U \), there is such open neighborhood \( V \) of point \( y \in Y \), that \( f^{-1}(V) \subset U \).
Proof. Necessity. Let the mapping \( f : uX \rightarrow vY \) be a \( z_u \)-closed and \( y \in Y \) be an arbitrary point and uniformly cozero-set \( U \in \mathcal{L}(uX) \), containing \( f^{-1}(y) \), i.e. \( f^{-1}(y) \subset U \). Then \( X \setminus U \in \mathfrak{F}(uX) \) is uniformly zero-set and \( f(X \setminus U) \) is closed in \( Y \). Set \( V = Y \setminus f(X \setminus U) \) is open in \( Y \) and \( y \in V \), i.e. \( V \) is open neighborhood of point \( y \). The next calculations: \( f^{-1}(V) = f^{-1}(Y \setminus f(X \setminus U)) = X \setminus f^{-1}(f(X \setminus U)) \subset X \setminus f^{-1}(U) = U \) are provided, i.e. \( f^{-1}(V) \subset U \).

Sufficiency. Conversely, let the condition of theorem is provided: \( F \in \mathfrak{F}(uX) \) be an arbitrary uniformly zero-set. The set \( U = X \setminus f \in \mathcal{L}(uX) \) is uniformly zero-set and for any \( y \in Y \setminus f(F) \) we have \( f^{-1}(y) \subset X \setminus f^{-1}(f(F)) \subset X \setminus f \) is open in \( Y \). Then there is an open neighborhood \( V_y \) of point \( y \in Y \setminus f(F) \) such, that \( f^{-1}(V_y) \subset U \). Suppose \( V = \cup \{ V_y : y \in Y \setminus f(F) \} \). Then \( V \) is open in \( Y \) and \( Y \setminus f(F) \subset V \) and \( f^{-1}(V) \subset U \), i.e. \( f^{-1}(V) \cap F = \emptyset \). Then \( V \cap f(F) = \emptyset \), i.e. \( V \subset Y \setminus f(F) \). Consequently, \( f(F) = Y \setminus V \), i.e. the set \( f(F) \) is closed. The theorem is proved completely.

The next theorem demonstrates, when \( z_u \)-closeness of mappings implies \( u \)-closeness.

**Theorem 2.2.** If a mapping \( f : uX \rightarrow vY \) is \( z_u \)-closed and \( f^{-1}(y) \) is Lindelöf for any point \( y \in Y \), then the mapping \( f \) is \( u \)-closed.

**Proof.** Let \( y \in Y \) be an arbitrary point, \( f^{-1}(y) \) be a Lindelöf and \( U \) be an arbitrary open set, containing \( f^{-1}(y) \), i.e. \( f^{-1}(y) \subset U \). Family \( \mathcal{L}(uX) \) is a basis of topology of the uniform space \( uX \) ([2]), hence for any point \( x \in f^{-1}(y) \subset U \) there exists such uniformly cozero-set \( V_x \in \mathcal{L}(uX) \), which is the open neighborhood of \( x \), then \( x \in V_x \subset U \). Then the family \( \{ V_x : x \in f^{-1}(y) \} \) is open covering of Lindelöf space \( f^{-1}(y) \). Let \( \{ V_{x_n} : n \in \mathbb{N} \} \) be a countable subcovering. Since \( V_{x_n} \in \mathcal{L}(uX) \) for all \( n \in \mathbb{N} \), then \( V' = \cup \{ V_{x_n} : n \in \mathbb{N} \} \) is uniformly cozero-set ([2]) and \( f^{-1}(y) \subset V' \subset U \). By \( z_u \)-closeness of mapping \( f : uX \rightarrow vY \), there is such open neighborhood \( V \) of point \( y \in Y \), that \( f^{-1}(V) \subset V' \subset U \). Then, on one of the closed mappings criterion ([3]), it follows, that the mapping \( f : uX \rightarrow vY \) is \( u \)-closed. 

The theorem is proved.

**Corollary 2.1.** Let \( f : uX \rightarrow vY \) be a bicom pact \( u \)-continuous mapping, i.e. \( f^{-1}(y) \) is bicom pact for any point \( y \in Y \). Then the next conditions are equivalent:

1. \( f : uX \rightarrow vY \) is \( z_u \)-closed.
2. \( f : uX \rightarrow vY \) is \( u \)-closed.

**Proof.** (1 \(\Rightarrow\) 2). It follows immediately from the Theorem 2.2.

(2 \(\Rightarrow\) 1) It follows from the Proposition 2.2.

**Corollary 2.1.** allows to define a special perfect mappings.

**Definition 2.3.** A mapping \( f : uX \rightarrow vY \) is called \( u \)-perfect, if it is \( u \)-closed and bicom pact.

**Remark 2.1.** Obviously, every uniformly perfect mapping ([1]) \( f : uX \rightarrow vY \) is \( u \)-perfect, and every \( u \)-perfect mapping \( f : uX \rightarrow vY \) is perfect.

**Example 2.2.** Let \( X \) be a locally bicom pact Tychonoff space and \( aX \) its one-point Alexandroff bicom pactification. Let \( \mathcal{U}_f \) be a fine uniformity on \( X \), and \( \mathcal{U}_a \) be a minimal precompact uniformity on \( X \) (see [7], [6], Chapter II, Ex.10), then \( \mathcal{U}_a \subset \mathcal{U}_f \) and \( \mathcal{U}_a \neq \mathcal{U}_f \), as for the Samuel bicom pactifications \( s_{aX}(X, s_{\mathcal{U}_f}) \) and \( s_{aX}(X, s_{\mathcal{U}_a}) \), we have \( s_{aX}X = \beta X \) is a Stone-Čech bicom pactification and \( s_{aX}X = aX \) is the Alexandroff bicom pactification. Obviously, \( \beta X \neq aX \) (it is suppose that there is more than one uniformity on \( X \)). A identical mapping \( 1_x : \mathcal{U}_aX \rightarrow \mathcal{U}_f X \) is a topological homeomorphism, it is not \( u \)-continuous mapping. Thus, the class of perfect and closed mappings more wider than the class of \( u \)-perfect and \( u \)-closed mappings.
The next properties of \( u \)-continuous mappings of the uniform spaces are take place.

**Proposition 2.3.** A composition \( g \circ f : uX \to wZ \) of \( u \)-continuous mappings \( f : uX \to vY \) and \( g : vY \to wZ \) is \( u \)-continuous mapping.

**Proof.** Immediately follows from the definition of \( u \)-continuous mapping (Definition 1.2). □

**Theorem 2.3.** If a composition \( g \circ f : uX \to wZ \) of \( u \)-continuous mappings \( f : uX \to vY \) and \( g : vY \to wZ \) is \( z_u \)-closed mapping, then restriction \( g \mid_{f(X)} : v'f(X) \to wZ \), where \( V' = V \cap f(X) \), is \( z_u \)-closed mapping.

**Proof.** Let \( N \in \mathfrak{F}(v'f(X)) \), i.e. \( N \) is a uniformly closed in \( f(X) \). Then from the properties of the uniformly closed sets (\([6]\)) it is follows there such \( N' \in \mathfrak{F}(vY) \) exists, that \( N = N' \cap f(X) \). Then \( f^{-1}(N') \in \mathfrak{F}(uX) \) and \( g \circ f : uX \to wZ \) is \( z_u \)-closed mapping by the condition of the theorem. We have \( g \mid_{f(X)}(N) = g \mid_{f(X)}(N' \cap f(X)) = g(N' \cap f(X)) = (g \circ f)(f^{-1}(N')) \) and \( g \mid_{f(X)}(N) \) is closed in \( Z \). The theorem is proved. □

**Corollary 2.2.** If a composition \( g \circ f : uX \to wZ \) of \( u \)-continuous mappings \( f : uX \to vY \) and \( g : vY \to wZ \) is \( u \)-closed mapping, then restriction \( g \mid_{f(X)} : v'f(X) \to wZ \), where \( V' = V \cap f(X) \), is \( u \)-closed mapping.

**Proof.** Proof follows from the \( z_u \)-closeness of any \( u \)-closed mapping (Proposition 2.4.). □

**Proposition 2.4.** Let \( f : uX \to vY \) be \( u \)-continuous mapping and \( u'X' \) be a uniform subspace of \( uX \). Then restriction \( f \mid_{X'} : u'X' \to v'f(X') \), where \( V' = V \cap f(X') \), is \( u \)-continuous mapping too.

**Proof.** Let \( F \) be a uniformly closed set in \( f(X') \), i.e. \( F \in \mathfrak{F}(v'f(X')) \). Then there such function \( f \in C^+(v'f(X')) \) exists, that \( F = g^{-1}(0) \). By the Katetov Theorem ([7]) there such function \( h \in C^+(vY) \) exists, that \( h \mid_{f(X')} = g \). Then a function \( h \circ f : uX \to uR \) is \( u \)-continuous and \( (h \circ f) \mid_{X'} = g \circ f \mid_{X'} \). Hence we have \( (g \circ f)(X')^{-1}(0) = (h \circ f)(X')^{-1}(0) = f^{-1}(h^{-1}(0)) \cap X' = f^{-1}(g^{-1}(0)) \in \mathfrak{F}(u'X') \), where \( f^{-1}(h^{-1}(0)) \in \mathfrak{F}(uX) \) and \( f^{-1}(g^{-1}(0)) \cap X' = f^{-1}(h^{-1}(0)) \). The proposition is proved. □

**Proposition 2.5.** Let \( f : uX \to vY \) be \( z_u \)-closed mapping and \( v'Y' \) be a uniform subspace of \( vY \), where \( V' = V \cap Y' \) and \( Y' \subset Y \). Then a mapping of restriction \( f \mid_{f^{-1}(Y')} : u'f^{-1}(Y) \to v'Y' \), where \( U' = U \cap f^{-1}(Y') \), is \( z_u \)-closed mapping too.

**Proof.** It follows from the equality \( f \mid_{f^{-1}(Y')} \left( N \cap f^{-1}(Y') \right) = f(N) \cap Y' \) for any \( N \in \mathfrak{F}(X) \). Proposition is proved. □

**Proposition 2.6.** Let \( f : uX \to uY \) be \( u \)-closed mapping and \( u'X' \) be closed uniform subspace of \( uX \). Then a restriction \( f \mid_{X'} : u'X' \to v'f(X') \), where \( U' = U \cap f(X') \), is \( u \)-closed mapping too.

**Proof.** It follows from the Proposition 2.13. and definition of \( u \)-closed mappings. □

**Theorem 2.4.** Let \( f : uX \to vY \) and \( g : uX \to wZ \) be a surjective \( u \)-continuous mappings of the uniform spaces \( uX, vY, wZ \) and \( f \) is a \( u \)-closed mapping. Then diagonal product \( f \triangle g : uX \to v \times wY \times Z \), where \( V \times W \), is the product of the uniformity \( V \) and \( W \), is \( u \)-closed mapping.
Proof. For a diagonal mapping $f \triangle g : uX \to v \times wY \times Z$, by the definition, we have $(f \triangle g)(x) = (f(x), g(x))$. Let $i_X : uX \to uX$ and $i_Z : wZ \to wZ$ be identical uniform homeomorphisms. Suppose $f \times i_Z : u \times wX \times Z \to v \times wY \times Z$, $i_X \Delta g : uX \to u \times wX \times Z$, where $(f \times i_Z)(x, z) = (f(x), z)$ and $(i_X \Delta g)(x) = (x, g(x))$ for any $x \in X$ and $z \in Z$. If $F \subset X$ and $M \subset Z$ are closed sets, then $f(F) \times M$ is closed subset of $Y \times Z$, hence, $(f \times i_Z)(F \times M) = f(F) \times M$ and $f \times i_Z$ is $u$–closed mapping. The mapping $i_X \Delta g : X \times X \to X$ is uniform homeomorphism of the space $uX$ and $\Gamma_g = \{(x, g(x)) : x \in X\}$ is a graph of a mapping $g$, it is a closed subspace of $u \times wX \times Z$. The closeness of the graph $\Gamma_g$ in $X \times Z$ follows from the $uX$ and $wZ$ are Hausdorff spaces. Then mapping $f \triangle g$ is a composition of the mappings $i_X \Delta g : uX \to u \times wX \times Z$ and $f \times i_Z : vY \to v \times wY \times Z$, where $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$ and mapping $(f \times i_Z)|_{\Gamma_g}$ is $u$–closed as a restriction of the closed mapping $f \times i_Z$ onto the closed subspace $\Gamma_g \subset X \times Z$, and $i_X \Delta g : uX \to v \Delta Y$ is a uniform homeomorphism. Thus, $f \triangle g = (f \times i_Z)|_{\Gamma_g} \circ (i_X \Delta g)$ is a $u$–closed mapping. We have a diagram. The theorem is proved. \[\Box\]

**Theorem 2.5.** Let $f : uX \to vY$ and $g : vY \to wZ$ are a surjective $u$–continuous mappings of the uniform spaces $uX$, $vY$, $wZ$ and a composition $g \circ f : uX \to wZ$ is $u$–closed mapping. Then the mapping $f : X \to Y$ is $u$–closed too.

**Proof.** By the condition of theorem $g \circ f : uX \to wZ$ is $u$–closed and $f : uX \to vY$ is a $u$–continuous mapping, according to the Theorem 2.16., $f \triangle (g \circ f) : uX \to u \times wX \times Z$ is a $u$–closed mapping. By the surjectivity of mappings $f$ and $g \circ f$, we have $(f \triangle (g \circ f))(x) = (f(x), (g \circ f)(x))$ for any $x \in X$. Then $\{(f(x), (g \circ f)(x)) : x \in X\} = \{(f(x), g(f(x)) : x \in X\} = \{(y, g(y)) : y \in Y\} = \Gamma_g$.

Obviously, that $(f \triangle (g \circ f))(x) = \{(f(x), g(f(x)) : x \in X\} = \{(y, g(y)) : y \in Y\} = \Gamma_g$. The graph $\Gamma_g$ is closed subspace $Y \times Z$ and the mapping $\pi_Y|_{\Gamma_g} : v\Gamma_g \to vY$, where $\pi_Y : v \times wY \times Z \to vY$ and $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$, is uniform homeomorphic mapping. Then $f = \pi_Y|_{\Gamma_g} \circ (f \triangle (g \circ f)) : uX \to vY$ is $u$–closed mapping as a composition of the uniform homeomorphism $\pi_Y|_{\Gamma_g} : v\Gamma_g \to vY$ and $u$–closed mapping $f \triangle (g \circ f) : uX \to u \times wX \times Z$. The next diagram takes place. We note, that the closeness of graph $\Gamma_g$ in $Y \times Z$ is essential, as soon as for any closed $F \subset X$, $(f \triangle (g \circ f))(F) = F'$ is closed in $\Gamma_g$, hence it is closed in $Y \times Z$ and its image $\pi_Y|_{\Gamma_g}(F')$ is closed in $Y$. It means, that $f(F) = \pi_Y|_{\Gamma_g}(F')$ and $f(F)$ is closed in $Y$, i.e. $f$ is $u$–closed. The theorem is proved. \[\Box\]
References


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