TWMS J. Pure Appl. Math. V.6, N.1, 2015, pp.59-67

INTUITIONISTIC FUZZY IDEAL EXTENSIONS IN SEMIGROUPS

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Abstract. The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. The purpose of this paper is to introduce the concept of extension of an intuitionistic fuzzy ideal in semigroups.

Keywords: semigroup, intuitionistic fuzzy ideal, intuitionistic fuzzy completely prime (completely semiprime) ideal, intuitionistic fuzzy ideal extension.

AMS Subject Classification: 08A72, 20M12, 3F55.

1. Introduction

A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation [4]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. The concept of fuzzy sets was introduced by Lofti Zadeh [14] in his classic paper in 1965. Azirel Rosenfeld [10] used the idea of fuzzy set to introduce the notions of fuzzy subgroups. Nobuaki Kuroki [6, 7, 8] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki [6, 8]. In [7], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X.Y. Xie [12, 13], Y.B. Jun [5], are mentioned in the bibliography. X.Y. Xie [12] introduced the idea of extensions of fuzzy ideals in semigroups. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1, 2, 3] as a generalization of the notion of fuzzy sets. In this paper we introduce the notion of extension of intuitionistic fuzzy ideals in semigroups and observe some important properties.

2. Preliminaries

In this section we discuss some elementary definitions that we use in the sequel.

Definition 2.1. [9] If $(S, *)$ is a mathematical system such that $\forall a, b, c \in S$, $(a*b)*c = a*(b*c)$, then $*$ is called associative and $(S, *)$ is called a semigroup.

Definition 2.2. [9] A semigroup $(S, *)$ is said to be commutative if for all $a, b \in S$, $a * b = b * a$.

Throughout the paper unless otherwise stated S will denote a semigroup.

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Manuscript received August 2014.

Definition 2.3. [9] A left(right) ideal of a semigroup S is a non-empty subset I of S such that $SI \subseteq I$ ($IS \subseteq I$). If I is both a left and a right ideal of a semigroup S, then we say that I is an ideal of S.

Definition 2.4. [9] Let S be a semigroup. Then an ideal I of S is said to be (i) completely prime if $xy \in I$ implies that $x \in I$ or $y \in I \forall x, y \in S$, (ii) completely semiprime if $x^2 \in I$ implies that $x \in I \forall x \in S$.

Definition 2.5. [1, 2] The intuitionistic fuzzy sets defined on a non-empty set X as objects having the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Now we recall the following properties from [1, 2] on intuitionistic fuzzy sets:

Let A and B be two intuitionistic fuzzy subsets of a set X. Then the following expressions hold[1, 2]:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(iii) $A^C = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$,

(iv) $A \cap B = \{< x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\} >: x \in X\}$,

(v) $A \cup B = \{< x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\} >: x \in X\}$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$.

Definition 2.6. [11] Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of a set X, $\alpha \in [0, \inf\{\nu_A(x) : x \in X\}]$. An object having the form $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is called an intuitionistic fuzzy translation of A if $(\mu_A)^T_\alpha(x) = \mu_A(x) + \alpha$ and $(\nu_A)^T_\alpha(x) = \nu_A(x) - \alpha$ for all $x \in X$.

Definition 2.7. [11] Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of a set X, $\beta \in (0,1]$. An object having the form $A^M_\beta = ((\mu_A)^M_\beta, (\nu_A)^M_\beta)$ is called an intuitionistic fuzzy multiplication of A if $(\mu_A)^M_\beta(x) = \beta \cdot \mu_A(x)$ and $(\nu_A)^M_\beta(x) = \beta \cdot \nu_A(x)$ for all $x \in X$.

Definition 2.8. [11] Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of a set X, $\alpha \in [0, \inf\{\beta \cdot \nu_A(x) : x \in X\}]$, where $\beta \in (0,1]$. An object having the form $A^C_{\beta\alpha} = ((\mu_A)^C_{\beta\alpha}, (\nu_A)^C_{\beta\alpha})$ is called an intuitionistic fuzzy magnified translation of A if $(\mu_A)^C_{\beta\alpha}(x) = \beta \cdot \mu_A(x) + \alpha$ and $(\nu_A)^C_{\beta\alpha}(x) = \beta \cdot \nu_A(x) - \alpha$ for all $x \in X$.

Definition 2.9. A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semigroup S is called an intuitionistic fuzzy left(right) ideal of S if (i) $\mu_A(xy) \geq \mu_A(y)$(resp. $\mu_A(xy) \geq \mu_A(x)$) $\forall x, y \in S$, (ii) $\nu_A(xy) \leq \nu_A(y)$(resp. $\nu_A(xy) \leq \nu_A(x)$) $\forall x, y \in S$.

Definition 2.10. A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semigroup S is called an intuitionistic fuzzy two-sided ideal or an intuitionistic fuzzy ideal of S if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of S.

Definition 2.11. An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a semigroup S is called an intuitionistic fuzzy completely prime ideal of S if (i) $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$, (ii) $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$.

Definition 2.12. An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a semigroup S is called an intuitionistic fuzzy completely semiprime ideal of S if (i) $\mu_A(x) \geq \mu_A(x^2) \forall x \in S$, (ii) $\nu_A(x) \leq \nu_A(x^2) \forall x \in S$.

By routine verification we deduce the following theorem.

Theorem 2.1. *Let χ be the characteristic function of a non-empty subset A of S. Then A is an ideal(completely prime ideal, completely semiprime ideal) of S if and only if $(\chi, \overline{\chi})$ is an intuitionistic fuzzy ideal(resp. intuitionistic fuzzy completely prime ideal, intuitionistic fuzzy completely semiprime ideal) of S.*

3. Intuitionistic fuzzy ideal extensions

Definition 3.1. Let S be a semigroup, $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S and $x \in S$. The intuitionistic fuzzy subset $< x, A > = (< x, \mu_A >, < x, \nu_A >)$ where $< x, \mu_A >: S \to [0,1]$ and $< x, \nu_A >: S \to [0,1]$ is defined by $< x, \mu_A > (y) := \mu_A(xy)$ and $< x, \nu_A > (y) := \nu_A(xy)$ is called the intuitionistic fuzzy extension of $A = (\mu_A, \nu_A)$ by x.

Example 3.1. Let $X = \{1, \omega, \omega^2\}$, where ω is the cube root of unity. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X, defined as follows

$$\mu_A(x) = \begin{cases} 0.3 & \text{if } x = 1 \\ 0.1 & \text{if } x = \omega \\ 0.5 & \text{if } x = \omega^2 \end{cases} \text{ and } \nu_A(x) = \begin{cases} 0.4 & \text{if } x = 1 \\ 0.25 & \text{if } x = \omega \\ 0.3 & \text{if } x = \omega^2 \end{cases}.$$

Let $x = \omega$. Then the intuitionistic fuzzy extension of A is given by

$$< x, \mu_A > (y) = \begin{cases} 0.1 & \text{if } y = 1 \\ 0.5 & \text{if } y = \omega \\ 0.3 & \text{if } y = \omega^2 \end{cases} \text{ and } < x, \nu_A > (y) = \begin{cases} 0.25 & \text{if } y = 1 \\ 0.3 & \text{if } y = \omega \\ 0.4 & \text{if } y = \omega^2 \end{cases}.$$

The following properties follows easily.

Let A and B be two intuitionistic fuzzy subsets of a set X. Then the following expressions hold:

(i) $A \subseteq B$ if and only if $< x, A > \subseteq < x, B >$, (ii) $< x, A^C > = < x, A >^C$, (iii) $< x, A \cap B > = < x, A > \cap < x, B >$, (iv) $< x, A \cup B > = < x, A > \cup < x, B >$. (v) $< x, A >^C_{\beta\alpha} = < x, A^C_{\beta\alpha} >$.

If we put $\beta = 1$(respectively $\alpha = 0$) in intuitionistic fuzzy magnified translation then it reduces to intuitionistic fuzzy translation(respectively intuitionistic fuzzy multiplication). Consequently analogue of (v) follow easily in intuitionistic fuzzy translation and intuitionistic fuzzy multiplication.

We omit the proof of the following proposition.

Proposition 3.1. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of a commutative semigroup S and $x \in S$. Then $< x, A >$ is an intuitionistic fuzzy ideal of S.*

Remark 3.1. Commutativity of the semigroup S is not required to prove that $< x, A >$ is an intuitionistic fuzzy right ideal of S when $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy right ideal of S.

Definition 3.2. Let S be a semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S. Then we define $Supp\ A = \{x \in S : \mu_A(x) > 0 \text{ and } \nu_A(x) < 1\}$.

By routine calculation we can verify the following proposition.

Proposition 3.2. *Let S be a semigroup, $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S and $x \in S$. Then we have the following:*
(i) $A \subseteq < x, A >$.
(ii) $< x^n, A > \subseteq < x^{n+1}, A > \forall n \in N$.
(iii) If $\mu_A(x) > 0$ and $\nu_A(x) < 1$ then $Supp < x, A >= S$.

Definition 3.3. Let S be a semigroup, $A \subseteq S$ and $x \in S$. We define $< x, A >= \{y \in S : xy \in A\}$.

Now we can have the following proposition by routine calculation.

Proposition 3.3. *Let S be a semigroup and $\phi \neq A \subseteq S$. Then $< x, B >= C$ for every $x \in S$, where $B = (\chi_A, \overline{\chi}_A)$, $C = (\chi_{<x,A>}, \overline{\chi}_{<x,A>})$ are intuitionistic fuzzy subsets of S, χ_A and $\chi_{<x,A>}$ are characteristic functions of A and $< x, A >$ respectively.*

Proposition 3.4. *Let S be a commutative semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. Then $< x, A >$ is an intuitionistic fuzzy completely prime ideal of S $\forall x \in S$.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. Then by Proposition 3.1, $< x, A >$ is an intuitionistic fuzzy ideal of S. Let $y, z \in S$. Then

$$\begin{aligned} < x, \mu_A > (yz) &= \mu_A(xyz)(cf.\text{ Definition 3.1}) \\ &= \max\{\mu_A(x), \mu_A(yz)\}(cf.\text{ Definition 2.11}) \\ &= \max\{\mu_A(x), \max\{\mu_A(y), \mu_A(z)\}\} \\ &= \max\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_A(x), \mu_A(z)\}\} \\ &= \max\{\mu_A(xy), \mu_A(xz)\}(cf.\text{ Definition 2.11}) \\ &= \max\{< x, \mu_A > (y), < x, \mu_A > (z)\}(cf.\text{ Definition 3.1}) \end{aligned}$$

again

$$\begin{aligned} < x, \nu_A > (yz) &= \nu_A(xyz)(cf.\text{ Definition 3.1})) \\ &= \min\{\nu_A(x), \nu_A(yz)\}(cf.\text{ Definition 2.11}) \\ &= \min\{\nu_A(x), \min\{\nu_A(y), \nu_A(z)\}\} \\ &= \min\{\min\{\nu_A(x), \nu_A(y)\}, \min\{\nu_A(x), \nu_A(z)\}\} \\ &= \min\{\nu_A(xy), \nu_A(xz)\}(cf.\text{ Definition 2.11}) \\ &= \min\{< x, \nu_A > (y), < x, \nu_A > (z)\}(cf.\text{ Definition 3.1}). \end{aligned}$$

Hence $< x, A >$ is an intutionistic fuzzy completely prime ideal of S. $\square$

Remark 3.2. Let S be a semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. Then $< x, A >=< x^2, A >$.

Proposition 3.5. *Let S be a semigroup and $A = (\mu_A, \nu_A)$ be a non-empty intuitionistic fuzzy subset of S. Then for any $s, t \in [0, 1], < x, U(\mu_A; t) >= U(< x, \mu_A >; t)$ and $< x, L(\nu_A; s) >= L(< x, \nu_A >; s) \forall x \in S$.*

Proof. Let $y \in U(< x, \mu_A >; t)$. Then $< x, \mu_A > (y) \geq t$. This gives $\mu_A(xy) \geq t$ and hence $xy \in U(\mu_A; t)$. Consequently, $y \in < x, U(\mu_A; t) >$. It follows that $U(< x, \mu_A >; t) \subseteq < x, U(\mu_A; t) >$. Reversing the above argument we can deduce that $< x, U(\mu_A; t) > \subseteq U(< x, \mu_A >; t)$. Hence $< x, U(\mu_A; t) >= U(< x, \mu_A >; t)$.

Again let $z \in L(< x, \nu_A >; s)$. Then $< x, \nu_A > (z) \leq s$. This gives $\nu_A(xz) \leq s$ and hence $xz \in L(\nu_A; s)$. Consequently, $z \in < x, L(\nu_A; s) >$. It follows that $L(< x, \nu_A >; s) \subseteq < x, L(\nu_A; s) >$. Reversing the above argument we can deduce that $< x, L(\nu_A; s) > \subseteq L(< x, \nu_A >; s)$. Hence $< x, L(\nu_A; s) >= L(< x, \nu_A >; s)$. □

Proposition 3.6. *Let S be a commutative semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S such that $< x, A >= A$ for every $x \in S$. Then $A = (\mu_A, \nu_A)$ is a constant function.*

Proof. Let $x, y \in S$. Then by hypothesis we have $\mu_A(x) =< y, \mu_A > (x) = \mu_A(yx) = \mu_A(xy)$(since S is commutative) $=< x, \mu_A > (y) = \mu_A(y)$. By using similar argument we can show that $\nu_A(x) = \nu_A(y)$. Hence $A = (\mu_A, \nu_A)$ is a constant function. □

Corollary 3.1. Let S be a commutative semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. If A is not constant, A is not a maximal intuitionistic fuzzy completely prime ideal of S.

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. Then, by Proposition 3.4, for each $x \in S, < x, A >$ is an intuitionistic fuzzy completely prime ideal of S. Now by Proposition 3.2(i), $A \subseteq < x, A >$ for all $x \in S$. If $A =< x, A >$ for all $x \in S$ then by Proposition 3.6, A is constant which is not the case by hypothesis. Hence there exists $x \in S$ such that $A \subseteq < x, A >$ with $A \neq < x, A >$. This completes the proof. □

By routine verification we can prove the following.

Proposition 3.7. *Let S be a commutative semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely semiprime ideal of S. Then $< x, A >$ is an intuitionistic fuzzy completely semiprime ideal of S for all $x \in S$.*

Corollary 3.2. Let S be a commutative semigroup and $\{A_i\}_{i \in \Lambda}$ be a family of intuitionistic fuzzy completely semiprime ideals of S. Let $B = \bigcap_{i \in \Lambda} A_i$. Then for any $x \in S$, $< x, B >$ is an intuitionistic fuzzy completely semiprime ideal of S, provided B is non-empty.

Proof. Let $x, y \in S$. Then

$$\mu_B(xy) = \inf_{i \in \Lambda} \mu_{A_i}(xy) \geq \inf_{i \in \Lambda} \mu_{A_i}(x) = \mu_B(x)$$

and

$$\nu_B(xy) = \sup_{i \in \Lambda} \nu_{A_i}(xy) \leq \sup_{i \in \Lambda} \nu_{A_i}(x) = \nu_B(x).$$

Hence S being commutative semigroup, B is an intuitionistic fuzzy ideal of S.

Again let $a \in S$. Then

$$\mu_B(a) = \inf_{i \in \Lambda} \mu_{A_i}(a) \geq \inf_{i \in \Lambda} \mu_{A_i}(a^2) = \mu_B(a^2)$$

and

$$\nu_B(a) = \sup_{i \in \Lambda} \nu_{A_i}(a) \leq \sup_{i \in \Lambda} \nu_{A_i}(a^2) = \nu_B(a^2).$$

Consequently, $B = \bigcap_{i\in\Lambda} A_i$ is an intuitionistic fuzzy completely semiprime ideal of S. Hence by Proposition 3.7, $< x, B >$ is an intuitionistic fuzzy completely semiprime ideal of S. $\square$

Remark 3.3. The proof of the above corollary shows that in a semigroup the non-empty intersection of family of intuitionistic fuzzy completely semiprime ideals is an intuitionistic fuzzy completely semiprime ideal.

Corollary 3.3. Let S be a commutative semigroup, $\{S_i\}_{i\in I}$ be a non-empty family of completely semiprime ideals of S and $A := \bigcap_{i\in I} S_i \neq \phi$. Then $< x, B >$ is an intuitionistic fuzzy completely semiprime ideal of S for all $x \in S$, where $B = (\chi, \overline{\chi})$ is an intuitionistic fuzzy subset of of S and χ is the characteristic function of A.

Proof. By hypothesis $A \neq \phi$. For each element $x \in S$ let $x^2 \in A$. Then $x^2 \in S_i \forall i \in I$. Since each S_i is a completely semiprime ideal of S, then $x \in S_i \forall i \in I(cf.$ Definition 2.4). So $x \in \bigcap_{i\in I} S_i = A$. Hence A is a completely semiprime ideal of S. So $B = (\chi, \overline{\chi})$ is an intuitionistic fuzzy completely semiprime ideal of $S(cf.$ Theorem 2.1). Hence by Proposition 3.7, $< x, B >$ is an intuitionistic fuzzy completely semiprime ideal of S. $\square$

Theorem 3.1. *Let S be a semigroup. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy completely prime ideal of S and $x \in S$ such that $\mu_A(x) = \inf_{y\in S} \mu_A(y)$ and $\nu_A(x) = \sup_{y\in S} \nu_A(y)$. Then $< x, A >= A$. Conversely, if $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of S such that $< y, A >= A \forall y \in S$ with $\mu_A(y)$ not maximal in $\mu_A(S)$ and $\nu_A(y)$ is not minimal in $\nu_A(S)$, then $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy completely prime.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S and $x \in S$ be such that $\mu_A(x) = \inf_{y\in S} \mu_A(y)$ and $\nu_A(x) = \sup_{y\in S} \nu_A(y)$(it is to be noted that since each of $\mu_A(y)$ and $\nu_A(y) \in [0, 1]$, a closed and bounded subset of R, so $\inf_{y\in S} \mu_A(y)$ and $\sup_{y\in S} \nu_A(y)$ exists). Let $z \in S$. Then $\mu_A(x) \leq \mu_A(z)$ and $\nu_A(x) \geq \nu_A(z)$. Hence $\max\{\mu_A(x), \mu_A(z)\} = \mu_A(z)$..........$(i)$ and $\min\{\nu_A(x), \nu_A(z)\} = \nu_A(z)$.........$(ii)$. Then

$$< x, \mu_A > (z) = \mu_A(xz) = \max\{\mu_A(x), \mu_A(z)\}(\text{since } A \text{ is an intuitionistic fuzzy completely prime ideal of } S) = \mu_A(z)(\text{by using } (i)) \text{ and}$$

$$< x, \nu_A > (z) = \nu_A(xz) = \min\{\nu_A(x), \nu_A(z)\}(\text{since } A \text{ is an intuitionistic fuzzy completely prime ideal of } S) = \nu_A(z)(\text{by using } (ii)).$$

Consequently, $< x, A >= A$. Conversely, let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S such that $< y, A >= A \forall y \in S$ with $\mu_A(y)$ is not maximal in $\mu_A(S)$ and $\nu_A(y)$ is not minimal in $\nu_A(S)$. Let $p, q \in S$. Then

$\mu_A(pq) \geq \mu_A(p), \mu_A(pq) \geq \mu_A(q)$ and $\nu_A(pq) \leq \nu_A(p), \nu_A(pq) \leq \nu_A(q)$(since A is an intuitionistic fuzzy ideal of S)................(iii).

Now two cases may arise viz. Case-(i) : Either $\mu_A(p)$ or $\mu_A(q)$ is a maximal element of $\mu_A(S)$ and either $\nu_A(p)$ or $\nu_A(q)$ is a minimal element of $\nu_A(S)$.

Case-(ii) : Neither $\mu_A(p)$ nor $\mu_A(q)$ is a maximal element of $\mu_A(S)$ and neither $\nu_A(p)$ nor $\nu_A(q)$ is a minimal element of $\nu_A(S)$.

In Case-(i) without loss of generality, let us suppose that $\mu_A(p)$ be maximal in $\mu_A(S)$ and $\nu_A(p)$ be minimal in $\nu_A(S)$. Then $\mu_A(pq) \leq \mu_A(p)$ and $\nu_A(pq) \geq \nu_A(p)$. Consequently, $\mu_A(pq) =$

$\mu_A(p) = \max\{\mu_A(p), \mu_A(q)\}$ and $\nu_A(pq) = \nu_A(p) = \min\{\nu_A(p), \nu_A(q)\}$. In Case-$(ii)$, by hypothesis $< p, A >= A$ and $< q, A >= A$. Hence $< p, \mu_A > (q) = \mu_A(q) \Rightarrow \mu_A(pq) = \mu_A(q) = \max\{\mu_A(p), \mu_A(q)\}$ and $< p, \nu_A > (q) = \nu_A(q) \Rightarrow \nu_A(pq) = \nu_A(q) = \min\{\nu_A(p), \nu_A(q)\}$(using (iii)). Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy completely prime ideal of S. □

The following is the characterization theorem of a completely prime ideal of a semigroup which follows as a corollary to the above theorem.

Corollary 3.4. Let S be a semigroup and I be an ideal of S. Then I is a completely prime ideal of S if and only if for $x \in S$ with $x \notin I$, $< x, B >= B$, where $B = (\chi, \overline{\chi})$ is an intuitionistic fuzzy subset and χ is the characteristic function of I.

Proof. Let I be a completely prime ideal of S. Then, by Theorem 2.1, $B = (\chi, \overline{\chi})$ is an intuitionistic fuzzy completely prime ideal of S. For $x \in S$ such that $x \notin I$, we have $\chi(x) = 0 = \inf\limits_{y \in S} \chi(y)$ and $\overline{\chi}(x) = 0 = \sup\limits_{y \in S} \overline{\chi}(y)$. Then, by Theorem 3.1, $< x, B >= B$.

Conversely, let $< x, B >= B \forall x \in S$ with $x \notin I$. Let $y \in S$ be such that $\chi(y)$ is not a maximal element of $\chi(S)$ and $\overline{\chi}(y)$ is not a minimal element of $\overline{\chi}(S)$. Then $\chi(y) = 0$ and $\overline{\chi}(y) = 1$ and so $y \notin I$. So $< y, B >= B$. Hence by Theorem 3.1, $B = (\chi, \overline{\chi})$ is an intuitionistic fuzzy completely prime ideal of S. So I is a completely prime ideal of $S(cf.$ Theorem 2.1). □

To conclude the paper we state below the following results on intuitioninistic fuzzy magnified translation the proofs of which will follow similarly as their counterparts for intutionistic fuzzy subsets obtained above in view of the fact that intutionistic fuzzy magnified translation is also an intutionistic fuzzy subset.

Theorem 3.2. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of a commutative semigroup S and $x \in S$. Then $< x, A^C_{\beta\alpha} >= (< x, (\mu_A)^C_{\beta\alpha} >, < x, (\nu_A)^C_{\beta\alpha} >)$ is an intuitionistic fuzzy ideal of S.*

Theorem 3.3. *Let S be a commutative semigroup and $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely prime ideal of S. Then $< x, A^C_{\beta\alpha} >= (< x, (\mu_A)^C_{\beta\alpha} >, < x, (\nu_A)^C_{\beta\alpha} >)$ is an intuitionistic completely fuzzy prime ideal of S.*

Theorem 3.4. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy completely semiprime ideal of a commutative semigroup S and $x \in S$. Then $< x, A^C_{\beta\alpha} >= (< x, (\mu_A)^C_{\beta\alpha} >, < x, (\nu_A)^C_{\beta\alpha} >)$ is an intuitionistic fuzzy completely semiprime ideal of S.*

Remark 3.4. If we put $\beta = 1$(respectively $\alpha = 0$) in intuitionistic fuzzy magnified translation then it reduces to intuitionistic fuzzy translation(respectively intuitionistic fuzzy multiplication). Consequently analogues of Theorems 3.2-3.4 follow easily in intuitionistic fuzzy translation and intuitionistic fuzzy multiplication.

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