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BRIEF PAPER

ASYMPTOTIC METHOD FOR SOLUTION OF THE OPTIMIZATION PROBLEM WITH PERIODIC BOUNDARY CONDITIONS AND CONTROL IN GASLIFT PROCESS

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Abstract. In this paper the asymptotic method of solving the periodic boundary optimal control problem in gaslift is given. An example is presented to illustrate the obtained results.

Keywords: optimal control, asymptotic method, gaslift process.

AMS Subject Classification: 49J15, 49M25,49N10.

1. Introduction

In this work, the periodic boundary optimal control problem for the low-loss transmission of the mixture from the well bottom in the exploitation of the oil wells by gaslift method is considered [6]. Solution of this problem with periodic boundary condition using the asymptotic method is investigated and the solution algorithm is given to this problem [1, 2, 7]. The results are illustrated by example from practice.

2. Problem Statement

If to average small parameter describing lifting and movement of the gas and gas-liquid mixture in the ring-shaped space in the gas lift process on over the time [2, 5] then in the mathematical model we have the following non-linear differential equation

$$\dot{Q}(x,\mu)=\frac{2a\rho FQ^2(x,\mu)}{\mu c^2\rho^2F^2-Q^2(x,\mu)}, Q(0,\mu)=u_0(\mu)=u_0^{(0)}+\mu\cdot u_0^{(1)}+..., \tag{1}$$

here $a=\frac{g}{2\omega_c}+\frac{\lambda_c\omega_c}{4D}$; c is the sound speed; ω_c - speed of the mixture movement; ρ- gas density; λ_c-hydraulic resistance; g-free fall acceleration; F- effective area of the ring-shaped space and lift; $\mu=\varepsilon^2$- small parameter ; $\varepsilon=\frac{1}{2l}$; l- depth of the well and u- volume of injected gas. According to (2) we reguled the system by using the start control. The equation (1) due to coefficients (a_1, ρ_1, F_1, c_1) and (a_2, ρ_2, F_2, c_2) in the segments $0\le x\le\frac{1}{2}-0$ and $\frac{1}{2}+0\le x\le 1$ correspondently transforms into two different equations. These two equations are related to each other by following impuls system at the point $\frac{1}{2}$

$$Q(\frac{1}{2}+0,\mu)=\gamma Q(\frac{1}{2}-0,\mu)+(-\delta_3(Q(\frac{1}{2}-0,\mu))-\delta_2)^2+\delta_1)\bar{Q}, \tag{2}$$

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where γ, δ_1, δ_2, δ_3 are positive real numbers defined as in [2].

Here our aim is to make equal the volume of lifting GLM in the well bottom to its volume in the wellhead by injecting minimal gas [1]. Therefore, we add the following periodicity condition [4, 6]:

$$Q(\frac{1}{2}+0,\ \mu)=Q(1,\ \mu). \tag{3}$$

It is clear that this is not possible in practice. Therefore we take the condition (3) in the following form

$$Q(\frac{1}{2}+0,\ \mu)=\tau\cdot Q(1,\ \mu), \tag{4}$$

here τ is a positive real number in $0<\tau<1$. It is possible to achieve our aim by tending this parameter to 1. Thus, the problem consists of finding an initial gas volume $Q(0)$, which satisfies the conditions (1), (2) and minimizes the following function

$$J=\frac{1}{2}\alpha Q^2(\frac{1}{2},\mu)+\beta u^2(\mu), \tag{5}$$

where $\alpha<0$ and $\beta>0$ are real numbers.

For the solving of the problem the extended functional is written and then we get the Euler-Lagrange equation [3]

$$\dot{\lambda}(x,\mu)=-\frac{4ac^2\rho^3F^3Q(x,\mu)\cdot\mu}{(\mu c^2\rho^2F^2-Q^2(x,\mu))^2}\lambda(x,\mu) \tag{6}$$

and the following boundary conditions

$$\begin{gathered}\alpha Q(\frac{1}{2}+0,\mu)+(\frac{\beta}{2}-1)\cdot\lambda(\frac{1}{2}+0,\mu)-\frac{\beta}{2}\cdot\lambda(1)=0,\\ \gamma\lambda(\frac{1}{2}+0,\mu)-\frac{\beta}{2}\lambda(\frac{1}{2}-0,\mu)-2\delta_3\lambda(\frac{1}{2}+0,\mu)(Q(\frac{1}{2}-0,\mu)-\delta_2)\bar{Q}=0,\\ u(\mu)+\frac{\beta}{2}\lambda(0)+\delta(\mu)=0,\end{gathered} \tag{7}$$

where $\lambda(x)$ and δ are Lagrange multipliers. We search the solutions of the equations (1) and (6) in the form of following series

$$Q(x,\mu)=Q^{(0)}(x)+\mu Q^{(1)}(x)+\mu\frac{Q^{(2)}(x)}{2!}+...,\tag{8}$$

$$\lambda(x,\mu)=\lambda^{(0)}(x)+\mu\lambda^{(1)}(x)+\mu\frac{\lambda^{(2)}(x)}{2!}+...,\tag{9}$$

and $\delta(\mu)=\delta^0+\mu\cdot\delta^1+...$.

If to take into account these solutions in the equations (1) and (6) we obtain the following differential equations in the first approarch

$$\begin{gathered}\dot{Q}^{(0)}(x)=-2a\rho F, Q^{(0)}(0)=u_0^{(0)},\\ \dot{Q}^{(1)}(x)=-\frac{2a\rho^3F^3c^2}{Q^{(0)2}(x)}, Q^{(1)}(0)=u_0^{(1)}, ...,\\ \dot{\lambda}^{(0)}(x)=0, \dot{\lambda}^{(1)}(x)=-4c^2a\rho^3F^2Q^0(x)\cdot\lambda^0(x), ...\end{gathered}\tag{10}$$

Considering (8) and (9) in (7) the similar representations may be obtained for $Q^{(i)}(0),\ \lambda^{(i)}(0),\ \ Q^{(i)}(\frac{1}{2}-0), \lambda^{(i)}(\frac{1}{2}-0), \lambda^{(i)}(\frac{1}{2}+0), Q^{(i)}(\frac{1}{2}+0), \lambda^{(i)}(1), Q^{(i)}(1)$ and δ^i $i=0,1....$ If to linearize the conditions (7), then we obtain the following system of linear algebraic equations, which depends on the above variables

$$K_1 Z_1 = q_1, K_2 Z_2 = q_2, \tag{11}$$

where

$$Z_1 = \Big[Q^{(0)}(0), \lambda^{(0)}(0), Q^{(0)}(\frac{1}{2}-0), \lambda^{(0)}(\frac{1}{2}-0), Q^{(0)}(\frac{1}{2}+0),$$

$$\lambda^{(0)}(\frac{1}{2}+0), Q^{(0)}(1), \lambda^{(0)}(1), \delta^0\Big]^T,$$

$$K_1 = \begin{bmatrix} 0 & 0 & -(\gamma+2\delta_3\delta_2\bar{Q}) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & -1 & 0 & 0 & 0 \\ 0 & 0 & -2\delta_3\bar{Q} & 0 & 0 & \gamma & 0 & 0 & 0 \\ 1 & \frac{\beta}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\tau & 0 & 0 \end{bmatrix},$$

$$q_1 = \left[\delta_1\bar{Q} - 2\delta_3\delta_2^2\bar{Q}, 0, 0, -\delta, -a_1\rho_1F_1, 0, -a_2\rho_2F_2, 0, 0\right]^T,$$

$$K_2 = \begin{bmatrix} 0 & 0 & -2\delta_3\bar{Q}(Q^{(0)}(\frac{1}{2})-\delta_2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & \alpha & \frac{\beta}{2} & 0 & 0 & 0 \\ 0 & 0 & -2\delta_3\bar{Q}\lambda^{(0)}(\frac{1}{2}+0) & 0 & 0 & p & 0 & 0 & 0 \\ 1 & \frac{\beta}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -\tau & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Z_2 = \Big[Q^{(1)}(0), \lambda^{(1)}(0), Q^{(1)}(\frac{1}{2}-0), \lambda^{(1)}(\frac{1}{2}-0), Q^{(1)}(\frac{1}{2}+0),$$

$$\lambda^{(1)}(\frac{1}{2}+0), Q^{(1)}(1), \lambda^{(1)}(1), \delta^1\Big]^T,$$

$$q_2 = \left[0,\ \frac{\beta}{2}\lambda^{(0)}(1),\ \frac{\beta}{2}\lambda^{(0)}(\frac{1}{2}-0),\ 0,\ S,\ M,\ 0,\ W,\ G\ \right]^T$$

and 'T' is the operation of transpose.
Here

$$p = 1 - 2\delta_3 Q^{(0)}(\frac{1}{2}-0)\bar{Q},\ S = -\frac{2a_2\rho_2^3F_2^3c_2^2}{Q^{(0)2}(1)} + \frac{a_2\rho_2^3F_2^3c_2^2}{Q^{(0)2}(\frac{1}{2}+0)},$$

$$M = -4c_2^2a_2\rho_2^3F_2^2Q^{(0)}(1)\cdot\lambda^{(0)}(1)\ + 2c_2^2a_2\rho_2^3F_2^2Q^{(0)}(\frac{1}{2}+0)\cdot\lambda^{(0)}(\frac{1}{2}+0)$$

$$W = -\frac{a_1\rho_1^3 F_1^3 c_1^2}{Q^{(0)2}(\frac{1}{2}-0)},\ G = -2c_1^2 a_1 \rho_1^3 F_1^2 Q^{(0)}(\frac{1}{2}-0)\cdot\lambda^{(0)}(\frac{1}{2}-0).$$

If to take into account the last solutions$Q^{(0)}(0)$, $Q^{(1)}(0)$ found from (11) we can find the first approximation for the solutions $u \approx Q^{(0)}(0)+\mu\cdot Q^{(1)}(0)$ of the problem (1), (2), (5). The following algorithm is proposed for the solution of periodic problem by the asymptotical method.

3. Algoritm

- **Step 1.** Introduce the parameters a, ρ, F, c, l according to the intervals $0 < x < \frac{1}{2}-0$ and $\frac{1}{2}+0 < x < 1$
- **Step 2.** Input the matrices K_1,K_2 and the column vectors q_1, q_2 from (11).
- **Step 3.** Find $Q^{(0)}(0)$ and $Q^{(1)}(0)$ by solving the matrix algebraic equations $K_1Z_1 = q_1$ and $K_2Z_2 = q_2$ and define $Q(0) \approx Q^{(0)}(0)+\mu\cdot Q(^1)(0)+....$
- **Step 4.** Define $Q^{(0)}(x)$, $Q^{(1)}(x)$ from (11) and reconstruct $Q(x) \approx Q^{(0)}(x)+\mu\cdot Q^{(1)}(x)+...$.

Now we take the above parameters to calculate the corresponding and $Q(1,\ \mu)$ on the basis of the proposed algorithm with the optimal $Q(0)$ as $Q(l+0,\quad \mu) = 9,9231$,

Here is the plot of these calculations.

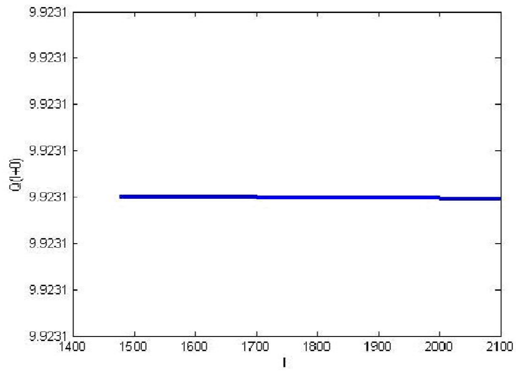

Fig. 1. Dependence of the mixture on the depth.

We see from the figure and the calculations that is 94.2518 % from . Thus, the model given in the work, allows one to lift the mixture from the well bottom to the wellhead 46% is increased to 94% as a implementation of the proposed model.

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