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STABILITY ESTIMATES FOR SOLUTIONS OF A LINEAR NEUTRAL STOCHASTIC EQUATION

IRADA A. DZHALLADOVA[1]

Abstract. A linear stochastic functional differential equation of neutral type is considered. Sufficient conditions for the exponential stability are derived by using Lyapunov-Krasovskii functionals of quadratic form with exponential factors. Upper bound estimates for the exponential rate of decay are derived in terms on the equation's coefficients.

Keywords: scalar linear stochastic differential delay equations of neutral type, asymptotic stability of zero solution, exponential stability and rate of decay, stochastic Lyapunov functionals.

AMS Subject Classification: 34K20, 34K40, 34K50, 34K6, 34K12.

1. Introduction

Theory and applications of functional differential equations form an important part of modern nonlinear dynamics. Those equations are natural mathematical models for various real life phenomena where the aftereffects are intrinsic features of their functioning. In recent years functional differential equations have been used to model processes in diverse areas such as population dynamics and ecology, physiology and medicine, economics and other natural sciences [6, 8, 12, 16]. In many of the models the initial data and parameters are subject to random perturbations, or the dynamical systems themselves represent stochastic processes. This leads to consideration of stochastic functional differential equations [7, 10, 17].

One of the principal problems of the corresponding mathematical analysis of equations is a comprehensive study of their global dynamics and related prediction of long term behaviors in applied models. Of those the problem of stability of a particular solution plays a significant role. The latter is typically reduced to the study of stability of the zero solution of a transformed system. In cases of complex systems the study of local stability is usually approached by considering the corresponding system linearized about the zero solution. Therefore, the study of stability of linear equations represents the first natural and important step in the analysis of more complex nonlinear systems.

When applying the mathematical theory to real life problems the mere statement about the stability in the system is hardly sufficient. In addition to the fact of the stability itself it is of significant importance to obtain constructive and verifiable estimates on the rate of convergence of solutions in time. One of the principal available tools in the related studies is the second Lyapunov method. For the functional differential equations the method has been developing in recent years in two main directions. The first one is the method of finite Lyapunov functions with the additional assumption of Razhumikhin [18]. The second one is the method of Lyapunov-Krasovskii functionals [14, 15]. For the stochastic functional differential equations some aspects of these two directions of research have been developed in papers [1]-[4], [11, 13, 14]

[1]Kiev National University of Economics, Kiev, Ukraine,
e-mail: irada-05@mail.ru
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and [5, 9, 13, 14, 19, 20], respectively. In the present paper, by using the method of Lyapunov-Krasovskii functionals, we derive sufficient conditions for stability together with the rate of convergence to zero of solutions for a class of linear the stochastic functional differential equation of neutral type.

2. Main results

Consider the following linear stochastic differential-difference equation of neutral type

$$d[x(t) - c\,x(t-\tau)] = [a_0 x(t) + a_1 x(t-\tau)]\,dt + [b_0 x(t) + b_1 x(t-\tau)]\,dw(t), \tag{1}$$

where $x \in \mathbb{R}$, a_0, a_1, b_0, b_1, c are all real constants, $\tau > 0$ is a constant delay, and $w(t)$ is a standard scalar Wiener process with

$$M\{d\,w(t)\} = 0,\ M\{d\,w(t)^2\} = dt,\ M\{d\,w(t_1)\,d\,w(t_2), t_1 \neq t_2\} = 0.$$

An F_t-measurable random process $\{x(t) \equiv x(t,\omega)\}$ is called a solution of equation (1) if it satisfies with the probability one the following integral equation

$$\begin{aligned} x(t) &= c\,x(t-\tau) + [x(0) - cx(-\tau)] + \int_0^t [a_0 x(s) + a_1 x(s-\tau)]\,ds + \\ &+ \int_0^t [b_0 x(s) + b_1 x(s-\tau)]\,dw(s), \quad t \geq 0 \end{aligned}$$

and the initial conditions $x(t) = \varphi(t), x'(t) = \psi(t), t \in [-\tau, 0]$. Here and for the remainder of the paper we will be assuming that the initial functions φ and ψ are continuous random processes. Under those assumptions the solution to the corresponding initial value problem for equation (1) exists and is unique for all $t \geq 0$, up to its stochastic equivalent solution on the space (Ω, F, P) [20]

We shall make use of the following norms for solutions of equation (1)

$$||x(t)||_\tau := \max_{-\tau \leq s \leq 0}\{|x(t+s)|\}, \quad ||x(t)||^2_{\tau,\beta} := \int_{-\tau}^0 e^{\beta s}\, x^2(t+s)\,ds. \tag{2}$$

As it is well known, the zero solution of equation (1) is called stable in the square mean if for every $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that every solution x of equation (1) satisfies $M\{|x(t)|^2\} < \varepsilon$ provided the initial conditions $x(s) \equiv \varphi(s), x'(s) \equiv \psi(s), s \in [-\tau, 0]$ are such that $\max_{-\tau \leq s \leq 0} |\varphi(s)| < \delta$ and $\max_{-\tau \leq s \leq 0} |\psi(s)| < \delta$. If the zero solution is stable in the square mean and $\lim_{t \to +\infty} M\{|x(t)|^2\} = 0$ then it is called asymptotically stable in the square mean. If there exist positive constants N and γ such that the inequality holds

$$M\{||x(t)||^2_{\tau,\beta}\} < N\,||x(0)||^2_\tau\, e^{-\gamma t},$$

then the zero solution is called exponentially (γ, β)-integrally stable in the square mean.

In this paper we derive constructive estimates of the exponential (γ, β)-integrable stability in the square mean of the differential-difference equation with constant delay (1). We employ the method of stochastic Lyapunov-Krasovskii functionals. In the papers [13, 14, 19, 20] the functional is chosen to be of the form

$$V[x(t), t] = h[x(t) - cx(t-\tau)]^2 + g\int_{-\tau}^0 x^2(t+s)\,ds$$

where constants $h > 0$ and $g > 0$ are such that the total stochastic differential of the functional along solutions is negative definite. In the present paper we add exponential factors to the functional so that it has the following form

$$V[x(t), t] = e^{\gamma t} \left\{ h[x(t) - cx(t-\tau)]^2 + g \int_{-\tau}^{0} e^{\beta s} x^2(t+s)\, ds \right\},$$

where constants $h > 0, g > 0, \gamma > 0, \beta > 0$ are to be determined later. This allows us not only to derive sufficient conditions for the stability of the zero solution but also obtain coefficient estimates on the rate of the exponential decay of solutions. Since the functional is homogeneous in both h and g we can set $h = 1$. By changing variables in the integral by $t + s = \xi$ we obtain

$$V[x(t), t] = e^{\gamma t} \left\{ h[x(t) - cx(t-\tau)]^2 + g \int_{t-\tau}^{t} e^{-\beta(t-\xi)} x^2(\xi)\, d\xi \right\}. \tag{3}$$

By using the earlier introduced norms (2) the functional (3) allows for the following two-sided estimates

$$\begin{aligned} &g e^{\gamma t} M\{||x(t)||^2_{\tau,\beta}\} \le M\{V[x(t), t]\} \le e^{\gamma t}(1 + c^2) \times \\ &\times M\{[x^2(t) + x^2(t-\tau)]\} + g e^{\gamma t} M\{||x(t)||^2_{\tau,\beta}\}. \end{aligned} \tag{4}$$

Introduce the following notations

$$A := -2a_0 - b_0^2,\ B := -a_1 + a_0 c - b_0 b_1,\ C := 2a_1 c - b_1^2, \tag{5}$$

$$S = S(g, \beta, \gamma) := \begin{bmatrix} A - \gamma - g & B + \gamma c \\ B + \gamma c & g e^{-\beta\tau} - \gamma c^2 + C, \end{bmatrix} \tag{6}$$

and let $\lambda_{\min} = \lambda_{\min}(S(g, \beta, \gamma))$ be the smallest eigenvalue of the matrix $S = S(g, \beta, \gamma)$.

The following result establishes the stability and the rate of growth of the solutions of equation (1) as $t \to +\infty$.

Theorem 2.1. *Suppose there exist positive constants β, γ, g with $\beta > \gamma$ such that the following inequalities are satisfied*

$$\begin{aligned} \Delta_1 &= A - \gamma - g > 0, \\ \Delta_2 &= (A - \gamma - g)(g\, e^{-\beta\tau} - \gamma c^2 + C) - (B + \gamma c)^2 > 0, \end{aligned} \tag{7}$$

where constants $A, B,$ and C are defined by (5). Then the zero solution of equation (1) is exponentially (γ, β)-integral stable in the square mean. Moreover, every solution $x(t)$ satisfies the following convergence estimate for all $t \ge 0$

$$M\left\{||x(t)||^2_{\tau,\beta}\right\} \le \left[\sqrt{\frac{1+c^2}{g}}\,(|x(0)| + |x(-\tau)| + ||x(0)||_{\tau,\beta})\right] e^{-\theta(\beta,\gamma)t}, \tag{8}$$

where θ is defined by

$$\theta = \theta(g, \beta, \gamma) = \min\left\{\beta, \frac{\lambda_{\min}(S)}{1+c^2} + \gamma\right\}. \tag{9}$$

Proof. In order to arrive at conditions (7) and the rate of solutions' decay (8) we shall apply the method of Lyapunov-Krasovskii functionals to the one given by formula (3). By using the Ito formula we evaluate the stochastic differential of functional (3) as follows

$$d\,V[x(t),t] = \gamma e^{\gamma t}\left\{[x(t)-cx(t-\tau)]^2 + g\int\limits_{t-\tau}^{t} e^{-\beta(t-s)}x^2(s)\,ds\right\}dt +$$
$$+e^{\gamma t}\ \{2[x(t)-cx(t-\tau)]d\,[x(t)-cx(t-\tau)] + g[b_0x(t)+b_1x(t-\tau)]^2\,(dw)^2 +$$
$$+\,g\left[x^2(t) - e^{-\beta\tau}x^2(t-\tau) - \beta\int\limits_{t-\tau}^{t} e^{-\beta(t-s)}x^2(s)\,ds\right]\,dt\left.\right\}.$$

By taking the mathematical expectation we obtain

$$M\frac{d}{dt}\left\{V[x(t),t]\right\} = e^{\gamma t}M\{\gamma[x(t)-cx(t-\tau)]^2 + 2[x(t)-cx(t-\tau)]\times$$
$$\times[a_0x(t)+a_1x(t-\tau)] + [b_0x(t)+b_1x(t-\tau)]^2 +$$
$$+gx^2(t) - ge^{-\beta\gamma}x^2(t-\tau) - g(\beta-\gamma)\int\limits_{t-\tau}^{t} e^{-\beta(t-s)}x^2(s)\,ds\}.$$

By using notations (5) and (6) the last expression can be rewritten in the following vector-matrix form

$$\frac{d}{dt}M\left\{V[x(t),t]\right\} = -e^{-\gamma t}M\left\{(x(t),x(t-\tau))\times S(g,\beta,\gamma)\times\right.$$
$$\left.\times\begin{pmatrix} x(t)\\ x(t-\tau)\end{pmatrix}\right\} - g(\beta-\gamma)e^{\gamma t}M\left\{\int\limits_{t-\tau}^{t} e^{-\beta(t-s)}x^2(s)\,ds\right\}.$$

With the notations (5) and (6) and according to the Silvester criterion [20] assumptions (7) imply that matrix S is positive definite. Assume now (7) to hold. Then the smaller eigenvalue of matrix S is positive, that is

$$\lambda_{\min}(S) = \frac{1}{2}\left[\sigma - \sqrt{\sigma^2 - 4\Delta_2}\right] > 0,$$
$$\sigma = A + C - \gamma(1+c^2) + g(e^{-\beta\tau}-1),$$

and the full derivative of the mathematical expectation of the Lyapunov functional is negative definite. The zero solution of equation (1) is then asymptotically stable.

We shall show next that solutions of equation (1) decay exponentially by calculating the corresponding exponential rate. Choose the constants β and γ to satisfy $\beta - \gamma > 0$. Then the full derivative of the mathematical expectation for the Lyapuvov functional satisfies the inequality

$$\begin{aligned}\frac{d}{dt}M\{V[x(t),t]\} &\leq -e^{\gamma t}\lambda_{\min}M\{[x^2(t)+x^2(t-\tau)]\} - \\ &- g(\beta-\gamma)e^{\gamma t}M\{||x(t)||^2_{\tau,\beta}\}.\end{aligned} \tag{10}$$

Let us derive conditions that are necessary to impose on the coefficients of equation (1) and parameters of the Lyapunov-Krasovskii functional (3) in order for the following inequality to hold

$$\frac{d}{dt}M\{V[x(t),t]\} \leq -\zeta M\{V[x(t),t]\},\ \text{for some } \zeta > 0.$$

We use a sequence of the following calculations.

(1) Rewrite the right hand side of inequality (4) in the form

$$-e^{\gamma t}M\{||x(t)||^2_{\tau,\beta}\} \quad \le \quad -\frac{1}{g}M\{V[x(t),t]\} + e^{\gamma t}\frac{1+c^2}{g}M\{[x^2(t)+x^2(t-\tau)]\}$$

and substitute the latter into inequality (10). This results in

$$\frac{d}{dt}M\{V[x(t),t]\} \le -e^{\gamma t}\lambda_{\min}M\{[x^2(t)+x^2(t-\tau)]\} +$$
$$+g(\beta-\gamma)\left\{-\frac{1}{g}\,M\{V[x(t),t]\} + e^{\gamma t}\frac{1+c^2}{g}M\{[x^2(t)+x^2(t-\tau)]\}\right\}.$$

Or equivalently as

$$\begin{aligned}\frac{d}{dt}M\{V[x(t),t]\} \quad &\le \quad -(\beta-\gamma)M\{V[x(t),t]\} - e^{\gamma t}\left\{\lambda_{\min}(S)-\right.\\ &- \quad \left.(\beta-\gamma)(1+c^2)\right\}M\{[x^2(t)+x^2(t-\tau)]\}.\end{aligned}$$

If the equation's coefficients and the functional's parameters are such that $\lambda_{\min}(S) > (\beta-\gamma)$ $(1+c^2)$ then the following holds

$$\frac{d}{dt}M\{V[x(t),t]\} \le -\zeta M\{V[x(t),t]\}, \quad \text{where} \quad \zeta = \beta-\gamma. \tag{11}$$

(2) Rewrite the right hand side of inequality (4) in the form

$$\begin{aligned}-e^{\gamma t}M\{[x^2(t)+x^2(t-\tau)]\} \quad &\le \quad -\frac{1}{1+c^2}M\{V[x(t),t]\} +\\ &+ \quad e^{\gamma t}\frac{g}{1+c^2}M\{||x(t)||^2_{\tau,\beta}\}\end{aligned}$$

and substitute the latter again into inequality (10). This results in

$$\frac{d}{dt}M\{V[x(t),t]\} \le \lambda_{\min}(S)\left\{-\frac{1}{1+c^2}M\{V[x(t),t]\} + e^{\gamma t}\ \times\right.$$
$$\left.\times\frac{g}{1+c^2}M\{||x(t)||^2_{\tau,\beta}\} - e^{\gamma t}g(\beta-\gamma)M\{||x(t)||^2_{\tau,\beta}\}\right\}.$$

Or, equivalently, in the inequality

$$\begin{aligned}\frac{d}{dt}M\{V[x(t),t]\} \quad &\le \quad -\frac{\lambda_{\min}(S)}{1+c^2}V[x(t),t] -\\ &- \quad e^{\gamma t}g\left\{(\beta-\gamma) - \frac{\lambda_{\min}(S)}{1+c^2}\right\}||x(t)||^2_{\tau,\beta}.\end{aligned}$$

If the parameter involved are such that the inequality

$$(\beta-\gamma) > \frac{\lambda_{\min}(S[g,\beta,\gamma])}{1+c^2}$$

holds then we deduce

$$\frac{d}{dt}M\{V[x(t),t]\} \le -\zeta M\{V[x(t),t]\}, \text{ where } \zeta = \frac{\lambda_{\min}(S)}{1+c^2}. \tag{12}$$

By simultaneously solving both differential inequalities (11) and (12) we see that

$$M\{V[x(t),t]\} \le V[x(0),0]\,e^{-\zeta t}, \quad \text{with} \quad \zeta = \min\{\beta-\gamma, \frac{\lambda_{\min}(S)}{1+c^2}\}.$$

By integrating the two-sided inequality (4) for the Liapunov-Krasovskii functional V we obtain

$$e^{\gamma t}gM\{||x(t)||^2_{\tau,\beta}\} \le M\{V[x(t),t]\} \le V[x(0),0]\,e^{-\zeta t} \le$$
$$\le \left\{(1+c^2)[x^2(0)+x^2(\tau)] + g||x(0)||^2_{\tau,\beta}\right\}\,e^{-\zeta t}, \quad t \ge 0.$$

This results in the final estimate for the rate of decay of solutions given by the following inequality

$$\begin{aligned} M\{||x(t)||^2_{\tau,\beta}\} &\leq \left[\sqrt{\frac{1+c^2}{g}}\,(|x(0)|+|x(-\tau)|)+||x(0)||_{\tau,\beta}\right]\times \\ &\times \; e^{-\theta(g,\beta,\gamma)\,t}, \qquad t\geq 0, \end{aligned}$$

where $\theta(g,\beta,\gamma)=\zeta+\gamma$ is given by formula (9). □

Suppose that constants g,β,γ are such that the assumptions (7) are satisfied with the corresponding rate (8) of decay of solutions taking place. In the stability situation one now has a flexibility to choose the parameters g,β,γ. Consider here a choice for the parameters which can be viewed in a sense as the best one.

According to the Silvester criterion the inequalities (7) are sufficient for matrix S to be positive definite. We will seek for the value of parameter g which maximizes the value of of the determinant Δ_2. This can be viewed as the best case for the positive definiteness of matrix $S[g,\beta,\gamma]$ since its minimal positive eigenvalue is then the largest possible one.

A necessary condition for Δ_2 to have a local extreme value in g is

$$\begin{aligned} \frac{d\Delta_2}{dg} &= \frac{d}{dg}\left\{(A-\gamma-g)(ge^{-\beta\tau}-\gamma c^2+C)-(B+\gamma c^2)^2\right\}= \\ &= -2ge^{-\beta\tau}+[(A-\gamma)e^{-\beta\tau}+(\gamma c^2-C)]=0. \end{aligned}$$

This implies that the corresponding value of g is given by

$$g_0=\frac{1}{2}\left[(A-\gamma)+e^{\beta\tau}(\gamma c^2-C)\right].$$

Accordingly, the corresponding matrix $S[g,\beta,\gamma]$ has the form

$$S=\begin{bmatrix} \frac{1}{2}\left[(A-\gamma)+e^{\beta\tau}(\gamma c^2-C)\right] & B+\gamma c \\ B+\gamma c & \frac{1}{2}\left[(A-\gamma)+e^{\beta\tau}(\gamma c^2-C)\right] \end{bmatrix}.$$

The quantities σ and Δ_2 are given by

$$\begin{aligned} \sigma &= \frac{1}{2}(1+e^{-\beta\tau})\left[(A-\gamma)-e^{\beta\tau}(\gamma c^2-C)\right], \\ \Delta_2 &= \frac{1}{4}e^{-\beta\tau}\left[(A-\gamma)-e^{\beta\tau}(\gamma c^2-C)\right]^2-(B+\gamma c)^2. \end{aligned}$$

This implies that the smaller solution of the characteristic equation

$$\lambda^2-\sigma\lambda+\Delta_2=0$$

is given by

$$\begin{aligned} &\lambda_{\min}=\lambda_{\min}(S[g_0,\beta,\gamma])=\frac{1}{2}\left[\sigma-\sqrt{\sigma^2-4\Delta_2}\right]= \\ &=\frac{1}{4}\left\{(1+e^{-\beta\tau})\left[(A-\gamma)-e^{\beta\tau}(\gamma c^2-C)\right]-\right. \\ &\left.-\sqrt{(1-e^{-\beta\tau})^2\left[(A-\gamma)-e^{\beta\tau}(\gamma c^2-C)\right]^2+16(B+\gamma c)^2}\right\}. \end{aligned}$$

Therefore, the above calculation establishes the following result.

Theorem 2.2. *Suppose there exist constants $\beta>\gamma>0$ such that the following inequality is satisfied*

$$A-\gamma-e^{\beta\tau}(\gamma c^2-C)>2e^{\frac{1}{2}\beta\tau}|B+\gamma c|,$$

where constants A, B, and C are defined by (5). Then the zero solution of equation (1) is asymptotically stable in the square mean. Moreover, every solution $x(t)$ satisfies the following convergence estimate for all $t \geq 0$

$$M\left\{||x(t)||^2_{\tau,\beta}\right\} \leq \left[\sqrt{\frac{1+c^2}{g}}\left(|x(0)|+|x(-\tau)|\right)+||x(0)||_{\tau,\beta}\right] e^{-\theta(\beta,\gamma)t},$$

where $\theta(\beta, \gamma)$ and $\lambda_{\min}$ are given by

$$\theta(\beta,\gamma) = \min\left\{\beta, \frac{\lambda_{\min}}{1+c^2}+\gamma\right\},$$

$$\lambda_{\min} = \frac{1}{4}\Big\{(1+e^{-\beta\tau})[(A-\gamma)-e^{\beta\tau}(\gamma c^2 - C)]-$$
$$-\sqrt{(1+e^{-\beta\tau})^2[(A-\gamma)-e^{\beta\tau}(\gamma c^2-C)]^2+16(B+\gamma c)^2}\Big\}.$$

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Irada Dzhalladova was born in Azerbaijan Republic. She graduated from Faculty of Mechanics and Mathematics of Baku State University in 1989. She got Ph.D. and Doctor of Sciences degrees in 1994 and 2009 correspondently in Kiev National University by named T.G. Shevchenko. Since 2010 year she is a Professor and Chief of Department of Higher Mathematics at Kiev National Economical University, Kiev, Ukraine. She published more than 140 research articles and supervised 5 Ph.D. students. Her research interests include stochastic dynamical systems, mathematical models in economics and others.