

BOOK REVIEW

Probability, Entire Functions and Estimations of Wiman-Valiron Type for Evolution Equations

N.M. Suleymanov (Moscow, "Moscow University Press", 2012. - 235p., ISBN 978-5-211-06420-1, in Russian)

Reviewed by V.A. Il'in*

The monograph is devoted to establishing Wiman-Valiron type estimations for evolution equations in Hilbert space. In the monograph a new scientific direction in the field of quality theory of differential equations-theory of Wiman-Valiron type estimations was initiated. Here it is developed a new direction connected with establishment of Wiman-Valiron estimations for solutions of evolution equations.

At the beginning of the last century the Wiman-Valiron estimates were established for analytic functions, therefore they are valid for harmonic functions as well, i.e. for the solutions of the Laplace equation. There arises a question: are the appropriate analogies valid for other types of equations? The monograph answers the stated question. Here the Wiman-Valiron type estimations are established for a wide class of equations: for parabolic equations, for inverse-parabolic equations, for pseudoparabolic operators, for fundamental solution of the Cauchy problem, for some class of elliptic equations and so on. A new type estimations that are generally speaking, are fulfilled outside of exceptional sets of finite weighted measure, are established.

Such estimations were not known earlier.

For obtaining such estimations the author worked out a new method based on development of the method of American mathematician P.S. Rosenbloom with application of probability methods.

It should be noted that the results of the monograph can't be considered as a generalization of the earlier known estimations of Wiman-Valiron, since even in the case of entire functions here new results are obtained. In the present monograph a new direction in quality theory of partial equations is developed.

The Wiman-Valiron type estimations admit to get exact information on behavior of the problem solution near the singularities of initial or boundary conditions, and also establish deep analogy between the properties of analytic functions and properties of the solutions of evolution equations.

Since the initial data of the problem for parabolic equations may be distributions, here the obtained estimations may be applied to investigation of the Green functions of different boundary value problems in the vicinity of zero. As is known much prominence is given to this issue in theory of partial equations.

The model for this direction of investigations were the classic Wiman-Valiron theorems on connection of maximum of the modulus $M(r)$ and maximum term $\mu(r)$ of an entire function,

* Professor, Member of the Russian Academy of Sciences.

established at the beginning of the past century by Wiman and Valiron. The proof of this estimation is very difficult and bulky. Rosenbloom amplified and generalized this result for a positive function determined for large values of y .

In N.M. Suleymanov's monograph the Rosenbloom's method was essentially developed and improved by obtaining the estimations.

The proof method of the monograph's author, applied to analytic functions gives new results compared with ones that were obtained earlier by other mathematicians. It is shown in the monograph that using the author's method one can not only to amplify the results of Rosenbloom for entire functions, but also to establish their accuracy in the class of special functions. In fact these results of the author are the solutions of Rosenbloom's appropriate problems formulated by him among unsolved problems in his known paper. It should be noted that these last conclusions of the author play an essential role in obtaining by him new results of Wiman-Valiron type.

Note that the Wiman-Valiron type estimations for evolution equations were first established namely by N.M. Suleymanov. Therefore it is fair to call such estimations the Wiman-Valiron Suleymanov type estimations.

In conclusion in the monograph some unsolved problems on the subject of the monograph are formulated for independent research of readers.

It is seen from what has been said, in N.M. Suleymanov's monograph it was found a new perspective fruitful direction in the joint of such field of mathematics as quality theory of differential equations, probability theory, theory of analytical functions, spectral theory and others.