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NUMERICAL INVESTIGATION OF CONSTRAINED OPTIMIZATION OF TRANSIENT PROCESSES IN OIL PIPELINES

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Abstract. Optimal control problems are investigated to establishment the transient processes, when raw material over pipelines are investigated in the work. We carry out qualitative analysis concerning the dependence of the optimal transient period of the process on the dispersion coefficient and on the length of a pipeline section, on the quantity of the interval of admissible controls given on different classes of functions and for different values of initial and final steady-state regimes.

Keywords: oil pipeline, optimal transient period of process, unsteady motion, control actions, dispersion coefficient, range of admissible controls.

AMS Subject Classification: 49K, 49M, 65K, 65N.

1. Introduction

Problems of optimal control of transient processes met with in transporting hydrocarbon raw material over trunk pipelines when switching from one steady-state regime to another are investigated in the work. The processes of this kind take place when changing transportation regimes according to a plan and when a necessity of emergency shutdown of a pipeline [5, 1] occurs. The mathematical statement is described as a parametrical problem of optimal control of systems with distributed parameters. The time of a transient process is an optimized parameter. The value*s* of fluid consumption at the ends of a linear pipeline section serve as a control action*s*. Constraints are formed taking into account technological characteristics of pumping stations (pumps) and the conditions of pipeline strength. The proposed criterion of optimality reflects the fact of the impossibility to achieve complete stabilization of the regime (precise conditions of steady-state process) because of inaccurate operation of measuring devices. The considered problem is closely linked with the problem of control of wave processes, studied by a number of scientists (A.G.Butkovsky, V.A.Il'in, F.P.Vasil'ev, A.V.Borovsky, etc. [3, 4, 6, 7]). In contrast to the investigations carried out heretofore, in this work, we numerically investigate time-optimal problems with boundary control of the regimes of fluid (oil) transportation over pipelines under constraints of technological character imposed on control actions and on the state of a controlled object. We give a qualitative analysis of the dependence of the minimal time when the process steadies on dispersion coefficient (determined by hydraulic resistance coefficient, viscosity, and the diameter of a pipeline), on the length of a pipeline section, on the difference between the values of initial and final steady-state regimes, on the range of admissible controls for different values of initial and final steady-state regimes.

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2. Problem statement

Consider an isothermal process of the transportation of single-phase oil over linear section of a horizontal pipeline of length l, diameter d, and hydraulic resistance coefficient λ. The regime of fluid flow is assumed laminar; oil is assumed incompressible, having kinematic viscosity ν. At both ends of the oil pipeline, there are pumping stations providing given transmission regime.

Unsteady-state flow of an incompressible fluid for case of subsonic flow velocities is described by the following linearized system of differential equations [5]:

$$\begin{array}{l} -\frac{\partial \tilde{p}}{\partial \xi} = \rho\left(\frac{\partial \tilde{\omega}}{\partial \tau} + 2a\tilde{\omega}\right), \\ -\frac{\partial \tilde{p}}{\partial \tau} = c^2 \rho \frac{\partial \tilde{\omega}}{\partial \xi}, \end{array} \quad \xi \in (0, l), \qquad \tau > 0, \tag{1}$$

where $\tilde{p} = \tilde{p}(\xi, \tau)$ is fluid pressure, $\tilde{\omega} = \tilde{\omega}(\xi, \tau)$ is fluid flow velocity at the point of pipeline $\xi \in (0, l)$ at time instance $\tau > 0$; c is sound speed in the propagation medium; $\rho = const$ is fluid density, which is assumed constant for a dropping fluid; λ is hydraulic resistance coefficient; $2a = \frac{\lambda\tilde{\omega}}{2d} = \frac{64\nu}{\tilde{\omega}d}\frac{\tilde{\omega}}{2d} = \frac{32\nu}{d^2}$ is linearized friction coefficient. (Linearization is made taking into account that the regime of fluid flow is laminar. Under turbulent regime it is assumed that $2a = \lambda\tilde{\omega}_{cp}/2d$, where $\tilde{\omega}_{A@}$ is average value of the velocity of raw material pipeline transportation). In system (1), turn to dimensionless quantities [5], assuming that

$$p = \frac{\tilde{p}}{\tilde{p}_0}, \quad \omega = \frac{\tilde{\omega}}{\tilde{\omega}_0}, x = \frac{\xi}{l}, \quad t = \frac{c\tau}{l}, \ \tilde{p}_0 = c\rho_0\tilde{\omega}_0, \tag{2}$$

where $\rho_0 = const$ is fluid density; $\tilde{\omega}_0$ is any value of flow velocity typical for given problem; l is any typical length, for example, the length of a pipeline section.

Then, after some simple transformations we obtain a system with dimensionless variables:

$$\begin{array}{l} -\frac{\partial p}{\partial x} = \frac{\partial \omega}{\partial t} + \beta\,\omega, \\ -\frac{\partial p}{\partial t} = \frac{\partial \omega}{\partial x} \ , \end{array} \qquad \beta = \frac{2al}{c}, 0 < x < 1, t > t_0. \tag{3}$$

Suppose that till the point of time $t_0 = 0$ there was a steady-state regime in the pipeline defined by the conditions

$$\omega(x,t) \ = \ \omega_0 = const, \qquad x \in [0,\ 1], \quad t \le 0, \tag{4}$$

$$p(x,t) = p_0(x), \qquad x \in [0,\ 1], \quad t \le 0, \tag{5}$$

where known function $p_0(x)$ at given fluid flow velocity ω_0 is determined by geometrical dimensions of the pipeline and by the properties of the fluid (oil) itself by the formula obtained from (3) under the condition of steadiness:

$$p_0(x) = p_0(0) - \beta\,\omega_0\,x, x \in [0,\ 1]. \tag{6}$$

It is necessary to note that in practice, precise observance of the conditions of steadiness (4), (5) is impossible, as there are always minute disturbances in pipelines caused by certain irregularities in the operation of processing facilities resulting in comparatively small divergences from the conditions of steadiness (4), (5):

$$\begin{array}{l} |\omega(x,t) - \omega_0| \le \delta_\omega, \\ |p(x,t) - p_0(x)| \le \delta_p, \quad x \in (0,1), \quad t \le 0, \end{array} \tag{7}$$

where δ_ω, δ_p are given small positive values determined by portions or percentages of the values ω_0 and $p_0(x)$ of some steady-state regime, respectively.

In this connection a regime of raw material transportation over a pipeline will be called δ-steady-state regime if (7) is satisfied.

Conditions (5) and (6) hold due to operation of pumping stations that maintain the regime

$$\omega(0,t) = \omega(1,t) = \omega_0, \qquad t \le 0.$$

The problem of optimal control of transient processes consists in the following: it is necessary to switch the operation regime of the pipeline (4) and (5) to a new preset steady-state regime (8) and (9) in a minimal possible time.

$$\omega(x,t) = \omega_T = const, \qquad t \ge T\,,\ x \in [0,1]\,, \tag{8}$$

$$p(x,t) = p_T(x), \qquad t \ge T, \quad x \in [0,1], \tag{9}$$

where T is the time after which the new steady-state regime (8) and (9) proceeds.

Necessary change of the transit regimes in oil pipelines must be achieved by controlling the operation regimes of pumping stations, namely, due to the change of volume flow rate (which is equivalent to the change of raw material flow velocity) at the ends of a linear pipeline section

$$\omega(0,t) = u_1(t), \qquad \omega(l,t) = u_2(t), \quad t \in [0,T]\,, \tag{10}$$

provided that some technological and technical constraints are fulfilled:

$$\underline{u}_1 \le u_1(t) \le \bar{u}_1, \qquad \underline{u}_2 \le u_2(t) \le \bar{u}_2, \qquad t \in [0,T]\,, \tag{11}$$

where $u_1(t)\,,\ u_2(t)$ are piecewise constant functions.

When controlling real technological processes, including regimes of raw material pipeline transportation, the implementation of control actions on a class of piecewise continuous functions of time is often complicated or impossible. That is why, in practice, they consider control problems on technically easily implementable classes of functions such as piecewise constant, impulse, etc. [1]. In this connection in the work, we also consider a class of problems of boundary control of process (3) when control actions are piecewise constant functions of time of the form:

$$\begin{array}{l} u_i(t) = \mathrm{v}_{ij} = const,\ t \in [\tau_{ij-1}, \tau_{ij})\ , \qquad i = 1,2,\ j = \overline{1,L}\ , \\ \tau_{i0} = 0, \quad \tau_{iL} = T, \quad \tau_{ij} = \tau_{ij-1} + \Delta\tau_{ij}\,,\ i = 1,2,\ j = \overline{1,L-1}. \end{array} \tag{12}$$

In this case, the optimal control problem consists in determining L-dimensional vectors $\mathbf{v_1},\ \mathbf{v_2} \in R^L$. In regard to points of time τ_{ij} and, respectively, intervals of constancy of the controls $\Delta\tau_{ij}$, they can be determined in a number of ways. If the number of switchings L of the control actions is given, then switching times τ_{ij} may be defined in different ways, for example, reasoning from the condition of uniformity of the intervals: $\tau_{ij} = j * ({}^{T}\!/_{L})$, i.e. $\Delta\tau_{ij} = \Delta\tau = const$, $j = \overline{1,L}$; one can also optimize the moments of switching times τ_{ij}, $i = 1,2$, $j = \overline{1,L-1}$. In this case, $(4L-2)$-dimensional vector $(\mathbf{v},\tau)$ $=(\mathrm{v}_{11},...,\mathrm{v}_{1L},\ \mathrm{v}_{21},...,\mathrm{v}_{2L},\ \tau_{11},...,\ \tau_{1L-1},\tau_{21},...,\ \tau_{2L-1})$ is optimized in the control problem. We can also optimize the number of switchings L.

In this work, when solving the problem of control of transient processes, it is assumed that L is given, and the values v_{ij} of piecewise constant controls on the intervals of constancy $[\tau_{ij-1}, \tau_{ij})$, as well as the switching times τ_{ij}, $i = 1,2$, $j = \overline{1,L}$ of the controls are optimized.

Reasoning from the conditions of pipeline strength, it is necessary to observe the following technological constraints on the maximal value of pressure when the transportation process takes place all over the pipeline and throughout the period of control of the transient process:

$$\underline{p} \le p(x,t) \le \bar{p}, \qquad x \in (0,1), \qquad t \in [0,T]\,, \tag{13}$$

where $\bar{p}$ is given maximum admissible value of pressure, which depends on the properties of the material of pipelines; $\underline{p}$ is the pressure level below which undesirable cavitation (boil) process of oil takes place.

Constraints (13) can be transformed into constraints on maximum admissible values of linear velocity $\bar{\omega}$, which can be obtained using Zhukovsky's formula for hydraulic shock pressure [5]: $\Delta p = c\rho\Delta\omega$, where Δp is the shock increment of pressure in the fluid of density ρ when the fluid flow velocity changes by $\Delta\omega$. Thus, using above mentioned formulas, we can obtain the following constraints on the values of velocity:

$$\underline{\omega} \le \omega(x,t) \le \bar{\omega}, \quad x \in (0,1), \quad t \in [0,T]\,. \tag{14}$$

As the system of equations (3) is a hyperbolic type system, then when studying the transient process in the pipeline we can find analogy with transient process in a distributed oscillating system with dispersion that is to be switched to given stationary state for a minimally possible period. As it is mentioned above, this problem has been considered by a lot of authors. For example, F.P.Vasil'ev and his followers established the existence of boundary control for the case when $T \ge 2l$ with the help of duality principle for a wave equation [7].

V.A.Il'in and his followers in a series of works ([6, etc.]) gave the analysis of the problem of existence of boundary control, obtained criteria (conditions) of controllability, as well as established its explicit analytical form. A.G.Butkovsky [4] solved the problem of fastest damping of an oscillating system by means of method of finite control. A.V.Borovsky [3] derived a formula of boundary control for an arbitrary heterogeneous string for cases of conditional and unconditional controllability.

In the above mentioned works, they do not take into account constraints of technological character imposed on control actions and on the state of the controlled object, without which the control of real processes is impossible.

The objective of the work is to investigate the solution to an optimal control problem of transient processes taking into account operating and technological constraints on the values of physical parameters participating in the transient process.

We minimize a target functional that tracks mean-square deviation of the values of velocity and pressure functions from the preset values during a definite period after the moment of δ-steadiness of the process:

$$J(u,T) = T + \int\limits_{T}^{T+DT}\int\limits_{0}^{l} \left\{ r_1 \left[p(x,t) - p_T(x)\right]^2 + r_2 \left[\omega(x,t) - \omega_T\right]^2 \right\} dxdt +$$

$$+R_1 \int\limits_{0}^{T}\int_{0}^{l} \left[\max(0, \omega(x,t) - \overline{\omega})\right]^2 dxdt +$$

$$+R_2 \int\limits_{0}^{T}\int\limits_{0}^{l} \left[\max(0, -\omega(x,t) + \underline{\omega})\right]^2 dxdt \ \to \min\,. \tag{15}$$

Here DT is the preset length of the time interval, during which we observe the process and establish the presence of δ-steady-state regime; r_1, r_2 are given weighting coefficients, the values of which are determined by δ; δ is the required precision of obtaining steadiness conditions; R_1, R_2 are sufficiently large positive numbers defining the penalties for violating the constraints on the phase variable (14). The problem stated can be considered as time-optimal problem for a distributed system with given values of the functions of phase at the time of the process completion T, which is considered as an optimized parameter, and with control parameters in boundary conditions. To solve this problem, two approaches can be applied. According to the first approach, we can consider T as a parameter and use two-level optimization: at the upper level to determine optimal time of the transient process T^*, we apply any method of one-dimensional optimization; at the lower level with given current values of T to determine $J_T^* = J(u_T^*, T) = \min\limits_u J(u, T)$, we solve a problem of optimal control of a distributed system with fixed time. According to the second approach, T is considered as a component of the control, and to find its optimal value, we apply a procedure of simultaneous combined optimization of T and $u\,(t)$.

Below we give formulas for the components of the gradient of the functional on the optimized parameters: operation regimes of pumping stations $(u(t), (\mathbf{v}, \tau))$ and the process completion time T. The obtained formulas allow using efficient numerical first-order optimization methods to solve the problem.

3. Formulas for numerical solution to the problem

Using the method of variation of the optimized functions [8] and the parameter T, we can obtain necessary optimality conditions in optimal control problem (3)-(5), (10)-(15), containing the following formulas for the components of the gradient of the functional on the control parameters and on the time of the process completion:

$$grad_{u_1} J(u,T) = c\psi_2(0,t), 0 \le t \le T + DT,$$

$$grad_{u_2} J(u,T) = c\psi_2(l,t), 0 \le t \le T + DT, \tag{16}$$

$$grad_T\, J(u,T) = 1 + \int\limits_0^l r_1(\omega(x,T+DT) + \omega(x,T) - 2\omega_T)(\omega(x,T+DT) - \omega(x,T))dx +$$

$$+ \int\limits_0^l r_2(p(x,T+DT) + p(x,T) - 2p_T(x))(p(x,T+DT) - p(x,T))dx +$$

$$+ \int\limits_0^l \{R_1\left[\max(0, \omega(x,T) - \overline{\omega})\right]^2 + R_2\left[\max(0, -\omega(x,T) + \underline{\omega})\right]^2\}dx\,. \tag{17}$$

Here $\psi_1(x,t)$,$\psi_2(x,t)$ are the solutions to the following adjoint boundary problem:

$$\frac{\partial \psi_1}{\partial t} = \begin{cases} -c\frac{\partial \psi_2}{\partial x} + \beta\psi_1 - 2r_1(\omega(x,t) - \omega_T), & T \le t \le T + DT, \\ -c\frac{\partial \psi_2}{\partial x} + \beta\psi_1 - 2R_1\left[\max(0, \omega(x,t) - \overline{\omega})\right] + & \\ \qquad +2R_2\left[\max(0, -\omega(x,t) + \underline{\omega})\right]), & 0 \le t\ < T, \end{cases}$$

$$\frac{\partial \psi_2}{\partial t} = \begin{cases} -\frac{1}{c}\frac{\partial \psi_1}{\partial x} - 2r_2(p(x,t) - p_T(x)), & T \le t \le T + DT, \\ -\frac{1}{c}\frac{\partial \psi_1}{\partial x}, & 0 \le t\ < T, \end{cases} \tag{18}$$

$$\psi_1(0,t) = \psi_1(1,t) = 0, 0 \le t < T + DT,$$

$$\psi_1(x, T+DT) = 0, \quad \psi_2(x, T+DT) = 0\ , 0 \le x \le 1. \tag{19}$$

In the case of piecewise constant control actions (12) the gradient of the functional in the space of control parameters v_{ij} is determined by the formulas:

$$\frac{dJ}{d\mathrm{v}_{1j}} = c\int\limits_{\tau_{1j-1}}^{\tau_{1j}} \psi_2(0,t)dt, \frac{dJ}{d\mathrm{v}_{2j}} = c\int\limits_{\tau_{2j-1}}^{\tau_{2j}} \psi_2(1,t)dt, i = 1,2, j = \overline{1,L}. \tag{20}$$

When we need to optimize the intervals of constancy, i.e. the switching moments of the control τ_{ij}, $i = 1, 2$, $j = \overline{1, L-1}$, then the gradient of the functional on the switching moments is determined by the formulas:

$$\frac{dJ}{d\tau_{1j}} = c\psi_2(0, \tau_{1j})(\mathrm{v}_{1j} - \mathrm{v}_{1j+1}), \frac{dJ}{d\tau_{2j}} = c\psi_2(1, \tau_{2j})(\mathrm{v}_{2j} - \mathrm{v}_{2j+1}), j = \overline{1, L-1}. \tag{21}$$

These formulas allow to use efficient numerical first order optimization methods for the solution to optimal control problem (3)-(5), (10)-(15), particularly, the methods of penalty function, of gradient projection, of conjugate gradient, and their combination can be applied [8]. As it is evident from (15), to take into account the constraints on the phase state (14), we use exterior penalty method [8].

4. The results of numerical experiments

Numerous computational experiments have been carried out with the purpose of revealing regularities of qualitative character of the dependence of the minimal time of the transient process of the length of a pipeline section, of dispersion coefficient a, and of the quantity of the difference between the values of the parameters of initial and of final steady-state regimes. The investigation of the problem of control of transient processes has been carried out under constraints on the control actions and on functions of phase (11), (14).

In all the numerical experiments, the control is implemented at the left end (the beginning of the section); boundary condition corresponding to a new required steady-state regime is established at the other end.

In the presence of technological constraints on the state distribution function and on control, we investigate the dependence of the minimal time when the transient process steadies from the number of intervals of admissible controls for different values of initial and final steady-state regimes.

When carrying out numerical computations, we assume that control actions belong to a class of piecewise constant and piecewise continuous functions.

The results of numerical experiments on the investigation of transient processes given below have been obtained under the following values of technological and technical parameters of oil transportation pipeline section: the internal diameter of the pipeline $d = 350mm$, oil density $\rho = 920kg/m^3$, kinematic viscosity coefficient $v = 1.5 \times 10^{-4}m^2/s$, sound speed in oil $c = 1200m/s$ (Note that these data correspond to actual oil pipeline data).

The description of the parameters (in dimensional units) of initial and final steady-state regimes of oil flow for the results of the solution to the problem of control of transient processes given below are shown in the table 1 (here q_0 is the value of fluid consumption under the initial steady-state regime; q_T is the value of fluid consumption under the final steady-state regime; $\tilde{p}(0, 0)$ is the pressure at the left end of the section under initial steady-state regime; $\tilde{p}(0, T)$ is the pressure at the left end of the section under final steady-state regime; l is the length of the linear pipeline section).

We have an evident link between the phase variable $\omega(x, t)$- the linear flow velocity – used in the description of transient processes and oil flow rate:

$$q(x, t) = \tilde{\omega}(x, t) \cdot S, q_0 = \tilde{\omega}_0 \cdot S, S = {}^{\pi d^2}\!/_4,$$

where $\tilde{\omega}_0$ is the feed flow velocity and q_0 the feed flow rate.

In order to switch to dimensionless variables in system (2) according to formulas (3), in problems *I-VII*, we scale $\tilde{\omega}(\xi, \tau)$ by taking the linear flow velocity $\tilde{\omega}_0$ as a characteristic quantity, corresponding to the flow rate q_0 under the initial steady-state regime, and in problems *VIII, IX*, we scale $\tilde{\omega}(\xi,\tau)$ by taking the linear flow velocity $\tilde{\omega}_T$ as a characteristic quantity, corresponding to the flow rate q_T under the final steady-state regime. We also make $\tilde{p}(\xi,\tau)$ dimensionless according to formulas (3). We take the values

$$\tilde{p}_0 = 6.182 * 10^5\, Pascal \approx 6.1\,\mathrm{atm}; \tilde{p}_0 = 4.637 * 10^5\,\mathrm{Pascal} \approx 4.6\,atm$$

as a scale of the pressure $\tilde{p}_0 = c\rho_0\tilde{\omega}_0$ for problems I-V and VI, respectively (for the other problems the values of $\tilde{p}_0$ are also determined by the respective values of $\tilde{\omega}_0$). In this case, transient processes in the problems considered are determined by the initial and final regimes (in dimensionless units) of the steady-state oil flow given in the table 1.

When taking into account the technological constraints on the control actions and on the system state, in contrast to the problem without constraints, we reveal the dependence of the optimal time of the transient process from the dispersion coefficient a. This dependence makes itself felt more when the interval of admissible values of the controls $[\underline{u}, \overline{u}]$ is narrowed. Demonstrate this fact by the example of the investigation of the problems III and IV, which differs only in the values of the dispersion coefficient (in the problem III a=0.0096, in the problem IV a=0.0144).

Compare the results of the solution to the problems III and IV, given in fig.1, under the same intervals of admissible values of the control actions (the results of the solution to the problems are given in dimensionless units on all the given figures). As it is evident from the figures, when we increase a the transient period decreases. Here we used the following notations: T_{opt}^{dml} is the optimal time of the transient process in dimensionless units; T_{opt}^{dm} the real optimal value of the transition time translated into seconds.

When investigating the dependence of the minimal transient period from the length of the section under technological constraints, we revealed that in contrast to the problem without constraints this dependence is not directly proportional. Show this fact using the results of the investigation of the problems I and II, in which all of the values of technological parameters are the same, except for the length of the section, which is in problem *II* twice as large as that in problem *I*. As it is evident from the data given in the table 2, when we narrow the interval of admissible values of the control the optimal transient period of the transient process T_{opt}^{dm} increases much slower than the directly proportional dependence with the increase of the length of the section, and becomes directly proportional under significant increase of the range of admissible values of the controls.

When carrying out numerical experiments on the investigation of transient processes under constraints on the values of the functions of phase and on control actions, we revealed that the transient period of the optimal transient process depends on the interval of admissible values of controls $[\underline{u}, \overline{u}]$ for given values of the initial ω_0 and final ω_T of the steady-state regimes. We also revealed the regularity stating that under the transient process at which the values of the final steady-state regime are greater than those of the initial (for example, as in the problems I, II, V, and VI) the transient period of the process depends only on the quantity of upper maximum admissible values $\overline{u}$ for the values of admissible control actions.

The analysis of the results of the computational experiments carried out for the problem V (see table 2 and figures metricconverterProductID2, a2, a, 2, b, 2, c) shows that when upper admissible values $\overline{u}$ <2.7, the transient period exceeds the minimal; at that the values of the optimal control actions under which the transient process is implemented are uniquely determined for every particular $\overline{u}$. Under the values of upper admissible limit $\overline{u}$ ≥2.7, the optimal time of the transient process does not change anymore remaining minimal (T_{opt}^{dml}≈2). However, when

$\overline{u}$ increases, the behaviour of the optimal control itself may significantly deteriorate from the point of view of practical implementation as there appear significant oscillations of the control (figures 2,d, 2,e, metricconverterProductID2,f2,f).

Similar results have been obtained when investigating the problems VI (table 2) and VII (table 3).

Consider transient processes in which the switch from larger values of fluid flow rate of steady-state regimes to smaller ones is implemented. Investigate them by the example of the problems VIII and IX (figure 3). When carrying out numerous computational experiments, we established that under the transient process in which the values of the final steady-state regime is less than those of the initial one the transient period of the process depends only of the lower limits of admissible control actions $\underline{u}$.

By the example of the problem V, move on to the investigation of transient processes under the assumption that the control is given on a class of piecewise constant functions; at that we optimize not only the values of the controls v_j, $j = \overline{1, L}$, but the intervals of constancy $\Delta\tau_j$, $j = \overline{1, L-1}$, or the switchings moments of the control τ_j as well (see table 4). Suppose that L (the number of intervals of constancy of the control) is given (particularly, $L = 10$ in all the computations presented). In case if the intervals of constancy $\Delta\tau_j = \tau_{j+1} - \tau_j$, or the difference between the adjacent values of the controls are small quantities, i.e. $\Delta\tau_j < \Delta\tau_{\min}$, or $|\mathrm{v}_j - \mathrm{v}_{j+1}| < \varepsilon$, where $\Delta\tau_{\min}$, ε are sufficiently small (particularly, in this problem $\Delta\tau_{\min}$=0.001 and ε=0.01), then we can "stick together" some intervals of constancy of the control function and, therefore, reduce the total number of these intervals (i.e. reduce L).

First, we give the optimal control obtained when investigating the problem V on a class of piecewise constant control functions without any constraints on the functions of phase and of control (figure 4, b). As it is evident from comparison with the respective graph obtained for piecewise continuous control under the same conditions (figure metricconverterProductID4, a4, a), here we do not observe such strong oscillation of the function, although the transient period slightly increases (on figure metricconverterProductID4, a4, a, $T_{opt}^{dml}\approx 2$; on figure 4, b, $T_{opt}^{dml}\approx 3.4$).

As it is evident from the graphs (figure 5), under the values $\overline{u}$ <2.7 there is significant reduction of the number of intervals of constancy mainly due to the switchings moments of the controls being optimized (when $\overline{u}$ =1.8 the control takes place at two intervals, when $\overline{u}$ =2 – at three intervals, and when $\overline{u}$ =2.2 – at five intervals of constancy).

When $\overline{u}$ ≥2.7 (figures 5, e, metricconverterProductID5, f5, f) the picture significantly changes: the number of the intervals increases, the transient period becomes equal to the minimal ($T_{opt}^{dml}\approx 3.4$) and does not change anymore with the increase of the range of upper permissible level of $\boldsymbol{u}$ and in this case the transient period coincides with the transient period for the problem without constraints on the functions of phase and of control (figure 4, b).

5. Conclusive remarks

Some of the existing quantitative results obtained when carrying out numerical experiments are given in the tables and graphs. On the basis of the obtained results, we give a qualitative analysis with respect to the solution to the problem of control of transient processes in oil pipelines.

The class of control actions and constraints imposed on the controls has a significant influence on the transient period. The main conclusions which are obtained on the basis of the analysis of the results of numerical experiments are as follows:

(1) As it is well known from theoretical investigations (this is confirmed by the results of the numerical experiments carried out) the minimal period of the transient process under piecewise continuous control actions without any constraints on the control process does not depend on the diameter of the pipeline, coefficient of resistance, viscosity, oil density, and the values of initial and final steady-state regimes. The minimal period of the transient process does depend on the length of the pipeline. But the optimal regimes of the pumping stations obtained here are practically unimplementable (by the example of the problem V, due to the obtained negative values of the velocity and its large oscillation (see figure metricconverterProductID4, a4, a)).

(2) Under technological constraints on the range (boundary) of the control actions from the class of piecewise continuous functions, there take place the following facts.

   (a) The dispersion coefficient influences the period of the transient process. Namely, when the dispersion coefficient increases, the transient period decreases. Here, when the range of the set of admissible controls increases, the influence of the dispersion coefficient decreases.

   (b) The difference between the values of the parameters of initial and final steady-state regimes has an influence on the transient process. Namely, the period of the transient process gets larger when we need to increase the consumption. The influence of this difference on the transient period decreases when the range of the set of admissible values of the control actions gets larger.

   (c) With the increase of the length of the pipeline section, the period of the transient process increases significantly slower when the interval of admissible values of the control is narrowed; this change becomes directly proportional to the increase of the length of the section when the range of admissible values of the control actions gets larger.

   (d) The minimal period of the transient process is the same when we need to switch from smaller value of the regime to a larger one, and vice versa. Here, the optimal transition regimes themselves are symmetrical (see figures metricconverterProductID2, a2, a, and 3, b).

(3) When controlling the transient process on a class of piecewise constant functions under technological constraints on the regimes of the control all the qualitative characteristics 2.1-2.4, which are intrinsic to piecewise continuous regimes, are observed on this class too.

   (a) In comparison to piecewise continuous controls, in case of using piecewise constant actions we need a larger period for the transient process under the same initial values of the technological parameters and of constraints.

   (b) The increase of the intervals of constancy and of the range of admissible values of the control results in the decrease of the period of the transient process.

   (c) In case of optimizing the switching moments of the controls, with narrowing the range of admissible values of the control there takes place significant reduction of the number of the intervals of constancy of the controls.

   (d) The numerical experiments carried out for the problems I-IX show that when controlling the process of steadiness at the section of the oil pipeline at the expense of pumping stations set at both its ends, regardless of the class of control actions and of the range of admissible controls the transient period decreases twice in comparison with controlling the pumping station at one end.

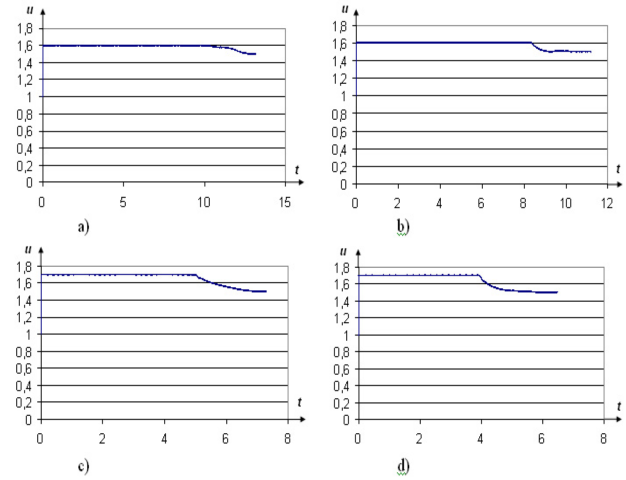

Figure 1.Graphs of optimal piecewise continuous controls for the problem III (left) and for the problem IV (right) with $\underline{u}$=0.5 and $\overline{u} = 1.6$, $a = 0.0096$ (a); $\overline{u} = 1.6$, $a = 0.0144$ (b); $\overline{u} = 1,7$, $a = 0.0096$ (c); $\overline{u} = 1,7$, $a = 0.0144$ (d).

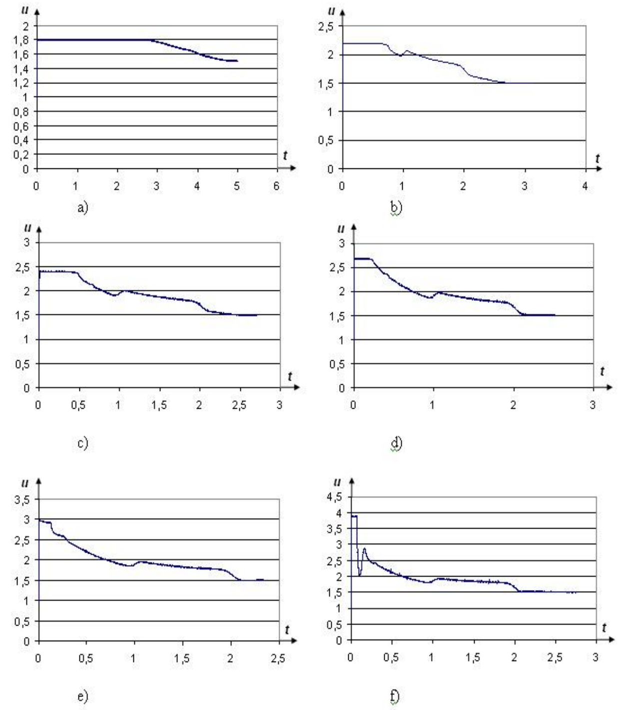

Figure 2.Graphs of optimal piecewise continuous controls for the problem V with $\overline{u} = 1.8$ (a), $\overline{u} = 2.2$ (b), $\overline{u} = 2.4$ (c), $\overline{u} = 2.7$ (d), $\overline{u} = 3$ (e), $\overline{u} = 4$ (f).

(e) It seems impossible to convert the mentioned above qualitative analysis to some quantitative estimations on the basis of computer-based experiments for arbitrary general case. But for each specific case of linear section of the oil pipeline and oil characteristics, we can obtain quantitative characteristics of transient processes and recommendations on how to control them in the form of graphs, tables, and more specific technological recommendations at the expense of carrying out numerous experiments.

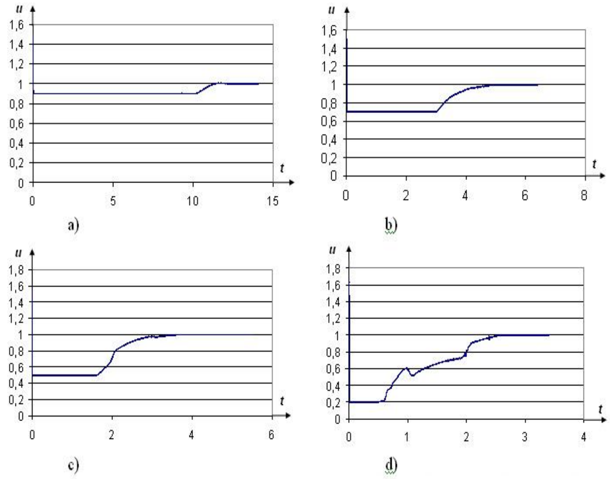

Figure 3.Graphs of optimal piecewise continuous controls for the problem $VIII$with $\underline{u}$=0.9 (a), $\underline{u}$=0.7 (b), $\underline{u}$=0.5 (c), $\underline{u}$=0.2 (d).

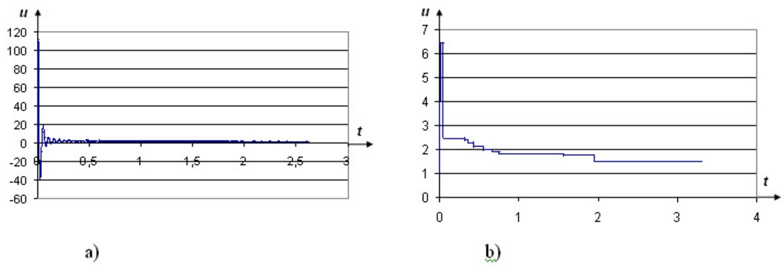

Figure 4.Graphs of control of the transient process for the problem V without constraints with piecewise continuous (a) and piecewise constant (b) controls.

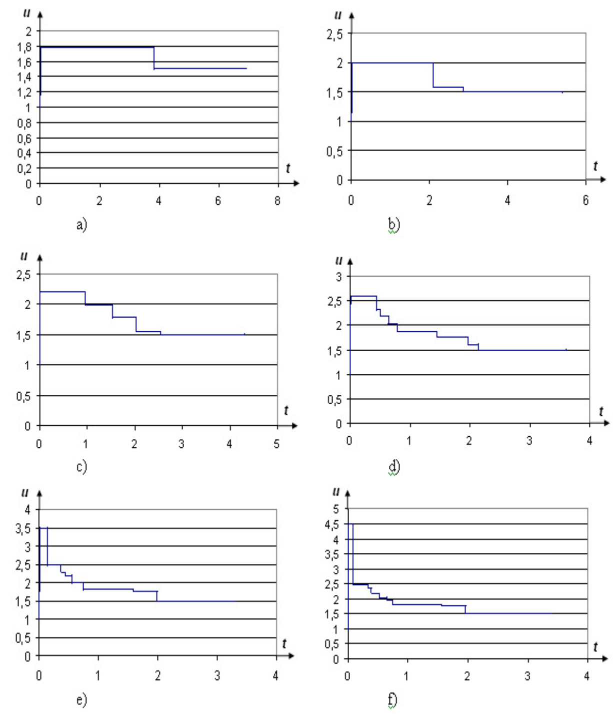

Figure 5.Graphs of piecewise constant optimal control for the problem V with $\overline{u}$=1.8 (a), $\overline{u}$=2 (b), $\overline{u}$=2.2 (c), $\overline{u}$=2.6 (d), $\overline{u}$=3.5 (e), $\overline{u}$=4.5 (f).

TABLE 1. Parameters of the considered problems of control of transient processes.

Problem No.	**In dimensional units**						**In dimensionless units**				
	$2a$ ($^1/_c$)	q_0 ($^{m^3}/_h$)	q_T ($^{m^3}/_h$)	$\tilde{p}(0,0)$ (atm.)	$\tilde{p}(0,T)$ (atm.)	l (km)	β	ω_0	ω_T	$p_0(0)$	$p_T(0)$
I	0.0192	400	600	30	43	132	2.112	1	1.5	4.8	7
II	0.0192	400	600	30	43	264	4.224	1	1.5	4.8	7
III	0.0192	400	600	23	33	132	2.112	1	1.5	3.7	5.3
IV	0.0288	400	600	23	33	132	3.168	1	1.5	3.7	5.3
V	0.0192	400	600	18	24	132	2.112	1	1.5	2.9	3.8
VI	0.0192	300	600	14	24	132	2.112	1	2	2.8	4.9
VII	0.0192	200	800	10	29	132	2.112	1	4	3.2	9.4
$VIII$	0.0192	600	400	24	18	132	2.112	1.5	1	3.8	2.9
IX	0.0192	600	300	24	14	132	2.112	2	1	4.9	2.8

TABLE 2. Results of the solution to the problems with $\underline{u}$=0.5 and with different values of $\overline{u}$ under piecewise continuous control actions.

$\overline{u}$	Problem I		Problem II		Problem V		Problem VI	
	T_{opt}^{dml}	T_{opt}^{dm}	T_{opt}^{dml}	T_{opt}^{dm}	T_{opt}^{dml}	T_{opt}^{dm}	T_{opt}^{dml}	T_{opt}^{dm}
1.6	17.9	1969	13.2	2904	12.7	1397		
1.7	9.6	1056	6.9	1518	6.9	759		
1.8	6.9	759	5.1	1122	4.8	528		
1.9	5.5	605	4.3	946	4.1	451		
2	4.6	495	3.7	814	3.4	374		
2.1	4.1	451	3.4	748	2.9	319		
2.2	3.7	407	3.3	726	2.7	297	7.3	803
2.3	3	374	3.1	682	2.6	286	5	550
2.4	3	330	3	660	2.5	275	4.2	462
2.5	2.8	308	2.9	638	2.4	264	3.8	418
2.6	2.7	297	2.7	594	2.3	253	3.1	341
2.7	2.6	286	2.7	594	2.1	231	2.7	297
2.8	2.5	275	2.6	572	2.1	231	2.6	286
3	2.4	264	2.5	550	2.1	231	2.4	264
3.2	2.4	264	2.4	528	2.1	231	2.2	242
3.4	2.3	253	2.3	506	2.1	231	2.1	231
3.6	2.2	242	2.2	484	2.1	231	2.1	231
3.7	2.1	231	2.1	462	2.1	231	2.1	231

TABLE 3. Dependence of the transient-process time from $\overline{u}$ in the problem VII with $\underline{u}$=0.5 for piecewise continuous controls.

$\overline{u}$	T_{opt}^{dml}	T_{opt}^{dm}	$\overline{u}$	T_{opt}^{dml}	T_{opt}^{dm}
4.2	16.6	1826	5.4	3.5	385
4.3	11.8	1430	5.6	3.1	341
4.4	9	990	5.8	2.7	297
4.5	7.5	825	6	2.6	286
4.6	6.5	715	6.3	2.5	275
4.7	5.8	638	6.6	2.4	264
4.8	4.9	539	7	2.3	253
4.9	4.6	506	7.4	2.2	242
5	4.3	473	7.6	2.1	231
5.2	4.1	451			

TABLE 4. Dependence of the transient-process time from $\overline{u}$ in the problem V with $\underline{u}$=0.5 for piecewise constant controls (τ_{L-1} is the time of the last switching of the control, L the number of intervals of constancy of the control).

$\overline{u}$	(τ_{L-1})	T_{opt}^{dml}	T_{opt}^{dm}	L
1.7	5.57	8.2	902	2
1.8	3.81	6.9	759	2
1.9	3.79	6.6	726	3
2	2.9	5.6	616	3
2.1	2.7	4.4	484	4
2.2	2.53	4.3	473	5
2.3	2.31	4.3	473	7
2.4	2.29	4.2	462	7
2.5	2.21	3.6	396	8
2.6	2.14	3.6	396	8
2.7	2	3.4	374	9 (10)

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