A METHOD TO DETERMINE THE COEFFICIENT OF HYDRAULIC RESISTANCE IN DIFFERENT AREAS OF PUMP-COMPRESSOR PIPES

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ABSTRACT. It is shown that in the oil production by the gas-lift method a basic problem is the determination of the coefficient of hydraulic resistance (CHR) on different areas in the pump-compressor pipes (PCP). Using a least-squares method and basing the analysis on the layer statistical data (or well histories, from the moment of starting of exploitation of the well to present), the computational algorithm is proposed to find the CHR on every given area of lift. An illustrative example is presented.

Keywords: ordinary nonlinear differential equation, gas lift, coefficient of hydraulic resistance, pump-compressor pipes.

AMS Subject Classification: 34A34, 76N15, 76N20.

1. INTRODUCTION

In exploitation of beds in oil production by a fountain, gas-lift methods, deep-well rod pumps, rodless pumps (centrifugal) [1, 6, 11], and also at transporting of oil products in collectors on distant distances from viscidity of carbohidrates takes place the paraffin deposite of inhibitors. Also, the presence of sand, corrosive agents and salts complicates a stream [11]. It requires the solution of the problem of determination CHR on PCP or on main pipelines [1, 12]. Such problems were considered in [1, 8, 9], in which averaged CHR was defined on all length of PCP or on pipelines between key points, i.e. on arcs [12]. However such approach doesn't allow to define the paraffin deposites (or CHR) on the certain required areas. For determination of CHR on the areas given beforehand as in [1, 2] will be based on the statistical data of fields. Using the least-quadratic method a quadratic objective functional is formed. Minimization of this functional on the CHR gives the required result. Note that this is a multi-parameter optimization problem (number of parameters $\lambda_i (i = \overline{1, N})$ are the number of given areas) is an extremely difficult problem [10]. Therefore, using the results of [1, 3], based on the modified gradient method, and the method of Gram-Schmidt orthogonalization, the computational algorithm for finding the CHR with high accuracy is given. The results are illustrated in a specific practical example.

2. PROBLEM STATEMENT

As it is known [4], the motion in a gas-lift process is described by the following ordinary nonlinear differential equation

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Manuscript received March 2016.

$$Q(x) = \frac{2a_i\rho_i F_i Q^2(x)}{c_i^2 \rho_i^2 F_i^2 - Q^2(x)} (i = 1, 2) \quad , \quad x \in [0, 2l],$$
(1)

with the initial conditions

$$Q\left(0\right) = Q_0,\tag{2}$$

where a_i , F_i, c_i , ρ_i , ω_i , λ_i , g_i (i = 1, 2) have the concrete practical values and are determined as in [4]; *l*-is the depth of the well or length of pumping compressor pipe. The solution of equations (1) in the interval [0, l-0), Q(l-0) is determined in the form

$$Q(l-0) = (-a_1\rho_1F_1l) + \frac{Q_0}{2} + \frac{c_1^2\rho_1^2F_1^2}{2Q_0} - \sqrt{\left(a_1\rho_1F_1(l) - \frac{Q_0}{2} - \frac{c_1^2\rho_1^2F_1^2}{2Q_0}\right) - c_1^2\rho_1^2F_1^2}$$
(3)

Transition from l-0 to l+0 in the shoe of the well, Q_{l+0} is determined by the following nonlinear impulse equation [5,7]

$$Q_{l+0} = F_{\delta}Q_{l-0} + \left[-\delta_3 \left(Q_{l-0} - \delta_2\right)^2 + \delta_1\right]\bar{Q},\tag{4}$$

where F_{δ} , δ_0, δ_1 , δ_2 is determined as in [8,9], \overline{Q} is an inflow from the layer to the bottomhole zone [5].

After determining Q_{l+0} by the (4) (i.e. forming gas liquid mixture GLM) the equation (1) is solved in the interval (l + 0, 2l]. Such approach of solving the problem (1), (2), (4) in the interval [0, 2l] yields $a_i = \frac{g_i}{2\omega_i} + \frac{2\lambda_i\omega_i}{4F_i}, i = 1, 2$, i.e. it is assumed that CHR is known from the well's history.

Determination of CHR in practice is a very intensive work, since the exploitation of the well is stopped. In [1, 2, 9] the determination of averaged CHR is investigated on all depth of lift. However, in practice usually the hydraulic resistance is different on the different areas of pipeline on all depth. Note that in gas-lift wells in PCP λ_c (hydraulic resistance) changes in an interval $1 \ge \lambda_c \ge 0$ [6]. We will suppose that after major repairs of the well on the different areas of PCP GLM is known - λ_i ($i = \overline{1, n}$). λ_1 corresponds to the beginning of the well, and λ_n to the end of PCP (terrene), here $\lambda_1 > \lambda_2 > ...\lambda_n$. Here λ_1 corresponds to the area($l + 0, l_1$], ..., and λ_n to the area ($l_{n-1}, 2l$]. From statistics also are known $Q_0^{j^{st}}$ [entrance of the well], $Q_{2l}^{j^{st}}$ (exit of the well) $j = \overline{1, m}$. Thus, solving the equations (1) with the initial condition $Q_0^{j^{st}}$ ($j = \overline{1, m}$) consistently in the intervals ($l + 0, l_1$],...,($l_{n-1}, 2 l$], $Q_{2l}^{j^{sol}}$ are found from (1). Thus, comparing the solutions of equation (1) $Q_{2l}^{j^{sol}}$ with $Q_{2l}^{j^{st}}$ it is possible to form the following inverse problem for determination of CHR on the different areas (the boundary of these areas are beforehand known or given) of PCP .

So, we will define the following quadratic functional

$$I = \frac{1}{2} \sum_{j=1}^{m} \left[Q_{2l}^{j^{st}} - Q_{2l}^{j} \left(Q_{0}^{j^{st}}, \lambda_{1}, \lambda_{2}, ..., \lambda_{n} \right) \right]^{2}$$
(5)

which is the difference of statistical data (debit of wells) and solutions of equations (1) with corresponding statistical initial conditions, that is the volume of injected gas at the beginning of annular space the given volume of gas. Here m is a number of statistical observations.

Thus, the problem consists of finding λ_i , $i = \overline{1, n}$, at which the functional (5) would get a minimum value.

3. Method of solution

Analytical calculating of gradient vector for $I(\lambda_1, \lambda_2, ..., \lambda_n)$ is practically impossible. Therefore, we calculate $\frac{\partial I}{\partial \lambda_i}$, $i = \overline{1, n}$ by the following formulas, which are partial derivatives with respect to the optimized variables λ_i , $i = \overline{1, n}$

where $\Delta_i, i = \overline{1, n}$ are enough small parameters. The parameters $\Delta_i, i = \overline{1, n}$ are searched so that an error in approximation would not exceed some defined limit. A minimum of functional (5) is as in works [1, 3].

Now we consider a more simple case, taking n = 2, i.e. the length of pipe is broken on two parts. We will suppose that the interval (l + 0; 2l) is split in two equal parts, i.e. in subintervals $[l+0; l_{ave}]$ and $[l_{ave}; 2l]$, where $l_{ave} = \frac{l}{2}$. We rewrite the quadratic functional (5) in a difference when length of PCP is broken more than on two parts in other form, because in this case for finding of the minimum of functional (5) a gradient can be obtained in an analytical form. Suppose that from history of trade for gas-lift wells at the beginning of annular space $Q_0^{j^{st}}$, and in the end of PCP $Q_{2l}^{j^{st}}$ are known. Also in both areas of PCP the coefficient of hydraulic resistances λ_1 and λ_2 accordingly in intervals $(l + 0; l_{ave}]$ and $(l_{ave}; 2l]$ are known. First at the set of initial conditions $Q_0^{j^{st}}(j=\overline{1,m})$ in the interval (0; l-0] one finds corresponding Q_{l-0}^j , after in (l-0; l+0) are found Q_{l+0}^i , and in an interval $(l+0; l_{ave})$ with initial values one finds the corresponding $Q_{lawe}^{j}\left(Q_{0}^{j^{st}},\lambda_{1}\right), \ j=\overline{1,m}$ in the following form

$$Q_{lave}^{j}\left(Q_{o}^{i^{st}},\lambda_{1}\right) = -a_{2}\rho_{2}F_{2} \cdot \frac{l}{2} + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)} - \sqrt{\left(a_{2}\rho_{2}F_{2} \cdot \frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2},$$

here $a_2 = \frac{g}{2\omega_{2c}} + \frac{\lambda_1\omega_{2c}}{4F_2}$. In the interval $[l_{ave}; 2l]$ in the end of PCP (at the point 2l, i.e. terrene) taking the statistical data $Q_{2l}^{j^{st}}$ as initial conditions, $Q_{l_{ave}}^{j}\left(Q_{2l}^{j^{st}},\lambda_{2}\right)$ are determined in the following form:

$$Q_{lave}^{j}\left(Q_{2l}^{j^{st}}, \lambda_{2}\right) = -a_{2}\rho_{2}F_{2} \cdot \left(-\frac{l}{2}\right) + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} - \sqrt{\left(a_{2}\rho_{2}F_{2} \cdot \left(-\frac{l}{2}\right) - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2}},$$

here $a_2 = \frac{g}{2\omega_2} + \frac{\lambda_2\omega_2}{4F_2}$. Thus for determining the CHR λ_1 and λ_2 in the PCP the functional will be in the form

$$I\left(Q_{0}^{j^{st}}, Q_{2l}^{j^{st}}, \lambda_{1}, \lambda_{2}\right) = \frac{1}{2} \sum_{j=1}^{N} \left[Q_{l_{ave}}^{j}\left(Q_{0}^{j^{st}}, \lambda_{1}\right) - Q_{l_{ave}}^{j}\left(Q_{2l}^{j^{st}}, \lambda_{2}\right)\right]^{2}$$
(6)

By minimizing the functional (6) on λ_1 and λ_2 can be determined the CHR in the PCP, and for this we must define the gradient of functional on the various λ_1 and λ_2 . Taking $a_2 = \frac{g}{2\omega_2} + \frac{\lambda_1\omega_2}{4F_2}$ the functional (6) get the form

$$I\left(Q_{0}^{j^{st}}, Q_{2l}^{j^{st}}, \lambda_{1}, \lambda_{2}\right) = \frac{1}{2} \sum_{j=1}^{m} \left[\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right) \rho_{2}F_{2}\frac{l}{2} + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)} - \frac{1}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j^{st}}\right)} - \frac{1}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j^{st}}\right)} - \frac{1}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j^{st}}\right)} - \frac{1}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j^{st}}\right)} - \frac{1}{2Q_{l+0}^{j^{st}}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j^{st}}\right)} - \frac{1}{2Q_{l+0}^{j^{st}}\left(Q_{0}^{j^{st}}\right)} - \frac{$$

So the functional (7) can be differentiated on λ_1, λ_2 analytically and the formula for the gradient $I(\lambda_1, \lambda_2)$ will be

$$\begin{split} \frac{\partial I\left(Q_{o}^{j^{st}},Q_{2l}^{j^{st}},\lambda_{1},\lambda_{2}\right)}{\partial\lambda_{1}} &= \sum_{j=1}^{m} \left[\left(\frac{\omega_{2}\rho_{2}l}{8} - \frac{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)\frac{\omega_{2}\rho_{2}l}{8} \right)}{\sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}}{4F_{2}}\right)\rho_{2}F_{2}l - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2}} \\ &\sum_{i=1}^{m} \left[\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2} + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)} - \frac{\sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)} - \frac{\sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2} - \frac{\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2}}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2} - \frac{\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2}}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2} - \frac{\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2}}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}},Q_{l-0}^{j}\right)}\right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2}} - \frac{\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}}\right)\rho_{2}F_{2}\frac{l}{2}} + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2}^{j}}{2Q_{2}^{j^{st}}}\right)}$$

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$$+ \sqrt{\left(\left(\frac{g}{2\omega_2} + \frac{\lambda_2\omega_2}{4F_2}\right)\rho_2 F_2 \frac{l}{2} - \frac{c_2^2 \rho_2^2 F_2 + Q_{2l}^{jst}}{2Q_{2l}^{jst}}\right)^2 - c^2 \rho^2 F^2}\right], \qquad (8)$$

$$\frac{\partial I\left(Q_{o}^{j^{st}}, Q_{2l}^{j^{st}}, \lambda_{1}, \lambda_{2}\right)}{\partial \lambda_{2}} = \sum_{j=1}^{m} \left[\left(-\frac{\omega_{2}\rho_{2}l}{8} + \frac{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{2}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} \right) \frac{\omega_{2c}\rho_{2}l}{8} \right)}{\sqrt{\left(\left(\frac{g}{\omega_{2}} + \frac{\lambda_{2}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} \right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2}} \\ \sum_{j=1}^{m} \left[\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} + \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)} - \frac{\sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{1}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)} \right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2} - \frac{\left(\frac{g}{2}\rho_{2}^{2}F_{2}^{2} + Q_{l+0}^{j^{2}}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)}{2Q_{l+0}^{j}\left(Q_{0}^{j^{st}}, Q_{l-0}^{j}\right)} \right)^{2} - c_{2}^{2}\rho_{2}^{2}F_{2}^{2} - \frac{\left(\frac{g}{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{2}} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} + \frac{\sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{2}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} \right)^{2} - c^{2}\rho^{2}F_{2}^{2}} \right]}{\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{2}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} + \sqrt{\left(\left(\frac{g}{2\omega_{2}} + \frac{\lambda_{2}\omega_{2}}{4F_{2}} \right) \rho_{2}F_{2}\frac{l}{2} - \frac{c_{2}^{2}\rho_{2}^{2}F_{2}^{2} + Q_{2l}^{j^{st}}}{2Q_{2l}^{j^{st}}} \right)^{2} - c^{2}\rho^{2}F_{2}^{2}F_{2}^{2}} \right]} \right]$$

Thus solving the systems of nonlinear algebraic equations (8), (9) relatively λ_1, λ_2 we find the CHR on two different areas of PCP.

Example. Let consider the following concrete example from practice [5, 11].

$$a_{1} = 0.101, \quad a_{2} = -89.77, \quad F_{\delta} = 1,$$

$$F_{1} = 0.006, \quad F_{2} = 0.004, \quad \delta_{1} = 0, 1,$$

$$c_{1} = 331, \quad c_{2} = 850, \quad \delta_{2} = 0, 01,$$

$$\rho_{1} = 0.717, \quad \rho_{2} = 700, \quad \delta_{3} = 0, 02.$$
(10)

here

$$\lambda_1 = 0.23105432 \ in \ (l+0, l_{ave}], \ and \ \lambda_2 = 0.10523117 \ in \ (l_{ave}, l_{2l}].$$
 (11)

Let the statistical data Q_0 and Q_{2l} have the following form¹

$$\begin{split} Q_0^{st} &= [0.0100; 0.0101; 0.0102; 0.2836; 0.2835; 0.2839; 0.2841; 0.2838; 0.5098; 0.5099; 0.5100] \\ Q_{2l}^{st} &= [3.461721010506153; 3.461737531819381; 3.461754049989395; 3.489037064136937; 3.489037027815357; 3.489037105930038; 3.489037077524699; 3.489037103252485; 3.424599378602579; 3.424496178515256; 3.424392782850191;] \end{split}$$

Then by using the model (1) (or (3), (4)) we will calculate Q_{l-0} , Q_{l+0} and Q_{lave} in the form

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¹These data are obtained from the concrete oil trades

$$\begin{split} Q_{l-0} &= [0.01006372359660190; 0.01016500864848524; 0.01026630661378647; \\ 0.3495746118174075; 0.3494193843315672; 0.3500404904979941 \\ 0.3503512399827695; 0.3498851648827839; 0.8669017889117823; \\ 0.8673012829091209; 0.8677011804606369] \end{split}$$

$$\begin{split} Q_{l+0} &= [9.993886655969986; 9.994024357306232; 9.994162035156325; \\ 10.22499963808979; 10.22499932577089; 10.22499999672104; \\ 10.22499975326095; 10.22499997362579; 9.690625081239599; \\ 9.689798765401157; 9.688970975499327] \end{split}$$

$$\begin{split} Q_{l_{ave}} &= [6.135009349265602; 6.135061240347568; 6.135113122058101; \\ 6.221331188920885; 6.221331073320471; 6.221331321692560; \\ 6.221331231528893; 6.221331313136034; 6.019373237388209; \\ 6.019054410455283; 6.018734994111583;] \end{split}$$

Solving the nonlinear algebraic equations (8), (9) by given parameters (10) we find $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, , which differ from the given λ_1 , λ_2 within 10^{-6} , and the functional has the following value

$$I(\lambda_1, \lambda_2) = 1.626303 \times 10^{-19},$$

and the first variations of (8), (10) will be

$$\frac{\partial I\left(Q_o^{j^{st}}, Q_{2l}^{j^{st}}, \lambda_1, \lambda_2\right)}{\partial \lambda_1} = -3.345831 \cdot 10^{-10}$$
$$\frac{\partial I\left(Q_o^{j^{st}}, Q_{2l}^{j^{st}}, \lambda_1, \lambda_2\right)}{\partial \lambda_2} = -7.10927 \cdot 10^{-11}$$

4. Conclusion

Thus in this case the problem is solved analytically, i.e. calculating the value of the gradient of the functional (5) the exacts formulas are obtained. However when the interval (l,2l) will be divided in more than two parts, such approach is not applicable. So here the quasilinearization method may be used to develop an iterative scheme for finding λ_i , $i \geq 3$.

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Navazi Ismailov, for a photograph and biography, see TWMS J. Pure Appl. Math., V.4, N.2, 2013, p.241



Hajan Hajiyev was born in 1946 in Georgia. He finished school from 1953. He graduated the Oil and Chemical Institute of Azerbaijan from 1969. He worked in different positions within different sections of Oil and Gas industry until 1996. He received his Ph.D. degree in 1982. Thereafter he began working as a senior research scientist in Scientific Research Institute of geotechnical problems in Oil, Gas and Chemicals industry. He is an author of over 80 scientific publications. His scientific work covers main directions of oil production.

M.F. Guliev, photo and biography were not presented.