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NUMERICAL SOLUTION OF A SYSTEM OF INDEPENDENT THREE-POINT DISCRETE EQUATIONS WITH NON-SEPARATED BOUNDARY CONDITIONS

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Abstract. In the paper a numerical approach to solving a system of three-point discrete equations with non-separated boundary conditions is proposed.The formulas for the solution of this problem are obtained and the algorithm for the application of the proposed method is given. The results of numerical solution of the problem, illustrating the effectiveness of the proposed method, are given.

Keywords: discrete equations, decomposition, complex object, non-separated boundary conditions, method transfer.

AMS Subject Classification: 49J15, 49J35.

1. Introduction

In this paper, we investigate the numerical solution of a system with a large number of independent three-point discrete equations with nonlocal boundary conditions unshared.

Many well-known mathematical models of discrete dynamic models of complex processes are obtained using the decomposition of complex objects into simpler subobjects with known mathematical models, or subobjects for which it's easy to build them ([15,18,19]). It is assumed that the decomposition of complex object carried out in such a way that the intermediate states of subobjects do not affect each other, and the relationship between subobjects is only through the initial and final states of the subobjects (see [7,11,14,20]). In reality, the subobjects usually associated with an arbitrary, but a small number of other subobjects and, therefore, the conditions defining the relationship between subobjectsare characterized by poorly filled Jacobi matrices (see [4, 5]).

In practice, this kind of problems arise in the numerical study of discrete models of complex processes in which their constituent sub-processes can be described, in particular, by the difference equations. In the general (nodal) points of the individual subobjects it's impossible to measure the value of each state variable subobject separately, but there are physical laws, which should satisfy the parameter values at the nodal points, which leads to indivisibility of setting the boundary conditions. In real life, such problems arise in the mathematical modeling of complex systems in electric power, unsteady motion of the liquid and gas pipeline systems with looped structure, adsorption and desorption processes of gases and others.

Direct use of the sweep method of boundary conditions for the solution of this type of problem is not effective, since it is possible to significantly speed up their decision, using the specific structure of condition setting.

In this paper a numerical approach based on the idea of the method of transfer of boundary conditions ([1-3,6,9]) for solving systems of independent discrete equations involving only the nodal points of the boundary conditions is proposed. The formulae for the implementation of the transfer conditions and the results of numerical experiments are obtained. As an illustration, consider the solution of the model problem arising in finite difference approximations of partial differential equations of hyperbolic type, which describes the motion of the fluid in the pipeline of complex loopback structure.

2. Statement of the problem

Consider a system of independent discrete second-order equations describing the complex discrete process (object), consisting of L mutually independent discrete subprocesses (subobjects):

$$a^{si} y^{si-1} + b^{si} y^{si} + c^{si} y^{si+1} = d^{si}, \; i = 2,...,N_s - 1, \; s = 1,...,L. \tag{1}$$

Here, the value y^{si} - defines the state of the $s-$process in the $i-$th discrete moment $i = 1,...,N_s$; $s = 1,...,L$; $a^{si}, b^{si}, c^{si}, d^{si}$ – given values, where a^{si}, b^{si}, c^{si} are nonzero values; N_s – the number of steps of the $s-$thsubprocess.

Considered independent discrete equations are linked by unshared boundary conditions of the form:

$$\sum_{s=1}^{L} v_1^{is} y^{s1} + \sum_{s=1}^{L} v_2^{is} y^{s2} = R_1^i, \; i = 1,...,l_1, \tag{2}$$

$$\sum_{s=1}^{L} w_1^{is} y^{sN_s-1} + \sum_{s=1}^{L} w_2^{is} y^{sN_s} = R_2^i, \; i = 1,...,l_2, \tag{3}$$

or more generally

$$\sum_{s=1}^{L} v_1^{is} y^{s1} + \sum_{s=1}^{L} v_2^{is} y^{s2} + \sum_{s=1}^{L} w_1^{is} y^{sN_s-1} + \sum_{s=1}^{L} w_2^{is} y^{sN_s} = R^i, \; i = 1,...,M \tag{4}$$

In practice, a setting of undivided boundary conditions both of the form (2), (3) and the conditions of the mixed nature of (4) can be found, of much importance is the fact that their total number in the assumption of their linear independence should be equal $2L$ (i.e., $l_1 + l_2 = 2L = M$).

The conditions (4) can be written in a more general form

$$V_1 y^1 + V_2 y^2 + W_1 y^{N-1} + W_2 y^N = R, \tag{5}$$

where we have introduced the following notations: V_1, V_2, W_1, W_2 – defined dimension matrices $M \times L$, $R = \left(R^1,...,R^M\right)^*$ – specified M –dimensional vector, $y^1 = (y^{11}, y^{21},...,y^{L1})^*$, $y^2 = (y^{12}, y^{22},...,y^{L2})^*$, $y^{N-1} = (y^{1N_1-1}, y^{2N_2-1},...,y^{LN_L-1})^*$, $y^N = (y^{1N_1}, y^{2N_2},...,y^{LN_L})^*$.

We assume that the problem is correct, i.e., solution is available and it is unique.

The relations (1) and (5) are mathematical models of many complex discrete functioning objects, processes with lumped or distributed parameters ([1-6]). For mathematical modeling of these processes has been used decomposition method for temporal and /or spatial variables, i.e. partitioning the entire object into separate subobjects whose function is independent from one another and can communicate between their input and output states, i.e. by the conditions (5).

To the problems of the form (1), (5) can also be reduced boundary value problems, described by a system of differential equations with ordinary or partial derivatives, which are used for solving the finite difference methods ([12,16,17]). In doing so, the system of equations consist of separate second order differential equations, linked only by the initial and / or boundary conditions. For this kind of boundary value problems leads, in particular, the study of the following tasks: calculation of branched electrical circuit using Kirchhoff's laws, when the knots and loops the circuit equates to the first and second Kirchhoff's law regarding the current and the voltage drop; calculation of complex pipeline ring, which is also produced using elektroanalogy Kirchhoff's laws, and for each node in a balance sheet expenses, and for each ring (loop) - the balance of pressures ([10,13]). The very process of motion on each linear section describes the hyperbolic system of two partial differential equations of the first order ([8,13]). After application of the method of lines for any of the variables of the problem data to the form (1), (5).

Among the features that characterize the mathematical models of many real complex structure of large objects, you can specify the following: 1) a large number of subobjects of L; 2) weak and arbitrary relationship between the subobjects, i.e. weak and arbitrary filling of matrices V_1, V_2, W_1, W_2; 3) longer duration of operation, i.e. a large number of steps $N_s, s = 1,...,L$ (individual for each sub-process).

Features 1), 3) for real objects lead to the fact that the order of the algebraic system (1), (5), equal to $L \times \sum_{s=1}^{L} N_s$, may exceed several thousand, which does not apply to their decision known numerical methods for solving systems of algebraic equations. Feature 2) leads to an undivided boundary conditions, which makes it necessary to use the methods of transfer of boundary conditions.

The aim of this work is to develop an efficient numerical method for solving a system of independent discrete equations (1) with undivided boundary conditions (5) with the specifications listed above. The method is based on an analog transfer method (sweep) conditions and reduced to solving a series of specially constructed discrete Cauchy problems on individual equations of the system (1).

3. Numerical solution to the problem

The approach proposed to solve the problem in question, based on the transfer of boundary conditions (5) in one end: to the left or right. This means that the relation (4) or (5) are replaced by their equivalents relations in which there will be no vectors y^1, y^2, when transferring conditions in the right end:

$$\tilde{W}_1 y^{N-1} + \tilde{W}_2 y^N = \tilde{R} \tag{6}$$

more details can be written as:

$$\sum_{s=1}^{L} \tilde{w}_1^{is} y^{sN_s-1} + \sum_{s=1}^{L} \tilde{w}_2^{is} y^{sN_s} = \tilde{R}^i,\ i = 1,\ldots,M . \tag{7}$$

When transferring conditions in the left end there will be no vectors y^{N-1}, y^N

$$\tilde{V}_1 y^1 + \tilde{V}_2 y^2 = \tilde{R} . \tag{8}$$

These conditions can be written as:

$$\sum_{s=1}^{L} \tilde{v}_1^{is} y^{s1} + \sum_{s=1}^{L} \tilde{v}_2^{is} y^{s2} = \tilde{R}^i,\ i = 1,\ldots,M . \tag{9}$$

After transferring conditions in one end we obtain (6), (7) or (8), (9), which constitute the system of M algebraic equations with unknowns y^1, y^2 or y^{N-1}, y^N. Solving these systems and determining y^1, y^2 or y^{N-1}, y^N solution of the task is achieved by performing simple calculations based on explicit recurrence formulas (Cauchy) with respect to certain discrete equations (1).

Selecting the direction of transport conditions (4) and (5) depends on the degree of filling of the matrices V_1, V_2, W_1, W_2 . Namely, when compared to the matrices W_1, W_2 , matrices V_1, V_2 are less filled ,then the conditions should be transferred to the right and vice versa. This rule will become apparent after the following description of the migration process.
Transfer conditions (5), more precisely (4), will be carried out for each of the $i-$th condition separately, $i = 1,\ldots,M$.

Thus, we consider an arbitrary $i-$th condition in (4), which after the transfer to the right takes the form (7), where $\tilde{w}_1^{is}$, $\tilde{w}_2^{is}$, $\tilde{R}^i$ – are yet unknown new coefficients. Getting the conditions in the form (7) will be carried out in stages.

Suppose that among the coefficients v_1^{is}, v_2^{is} , $s = 1,\ldots,L$, there are non-zero, otherwise the $i-$th condition is not necessarily transformed, because in this condition the only involved values are y^{sN_s-1}, y^{sN_s}. Let the first nonzero coefficient be $v_1^{ik} \neq 0$ or $v_2^{ik} \neq 0$, $1 \le k \le L$, $v_1^{is} = 0,\ v_2^{is} = 0$, for $s < k$.

Definition 1. We say that the values α^j, λ^j and β^j, $j = 1,...,N_k$ carry out the transfer of the $i-$th condition (4) with respect to $k-$th unknown y^{kj} to the right if for all vectors y^{kj} satisfying the equation (1), we have the equalities

$$\alpha^j y^{kj} + \lambda^j y^{kj+1} + \left[\sum_{s=k+1}^{L} (v_1^{is} y^{s1} + v_2^{is} y^{s2}) + \sum_{s=1}^{L} (w_1^{is} y^{sN_s-1} + w_2^{is} y^{sN_s}) \right] = \beta^j, \ j = 1,...,N_k . \quad (10)$$

It is clear that if $j = 1$ the following equalities should be satisfied:

$$\alpha^1 = v_1^{ik}, \ \lambda^1 = v_2^{ik}, \quad \beta^1 = R^i . \quad (11)$$

Values α^j, λ^j and β^j, $j = 1,...,N_k$ satisfying (10), (11) will be called sweep coefficients.

Substituting in (10) the values of the sweep coefficients, where $j = N_k$, we obtain new condition

$$\sum_{s=k+1}^{L} v_1^{is} y^{s1} + \sum_{s=k+1}^{L} v_2^{is} y^{s2} + \sum_{s=1}^{L} \tilde{w}_1^{is} y^{sN_s-1} + \sum_{s=1}^{L} \tilde{w}_2^{is} y^{sN_s} = \tilde{R}^i, \ i = 1,...,M ,$$

where we use the following notations

$$\tilde{w}_1^{ik} = w_1^{ik} + \alpha^{N_k}, \quad \tilde{w}_2^{ik} = w_2^{ik} + \lambda^{N_k}, \quad \tilde{w}_1^{is} = w_1^{is}, \ \tilde{w}_2^{is} = w_2^{is},$$

$s = 1,...,L, \ s \neq k, \ \tilde{R}^i = \beta^{N_k}$.

The right-wing sweep factors $\alpha^j, \lambda^j \ \beta^j$, producing the transfer conditions (4) to the right, can be define in different ways. One of them is suggested in the following theorem.

Theorem 1. Suppose that the values α^j, λ^j and β^j define by the following recurrent relations (discrete Cauchy problems):

$$\lambda^{j+1} = -\alpha^j \big/ \overline{a^k}, \ \alpha^1 = v_1^{ik}, \ \lambda^1 = v_2^{ik}, \quad j = 1,...,N_k$$

$$\alpha^{j+1} = \lambda^j + \lambda^{j+1} \overline{b^k} \quad (12)$$

$$\beta^{j+1} = \lambda^{j+1} \overline{d^k} + \beta^j, \ \beta^1 = R^i, \ j = 1,...,N_k .$$

Then $\alpha^j, \lambda^j, \beta^j$ are the right sweep coefficients for the $i-$th condition (4) with respect to the $k-$solution of equation (1).

Proof. Let's rewrite the system (1) in the form

$$a^{kj} y^{kj} + b^{kj} y^{kj+1} + c^{kj} y^{kj+2} = d^{kj}, \ j = 1,...,N_k - 2, \ k = 1,...,L .$$

from here

$$y^{kj+2} = \overline{d^k} - \overline{b^k} y^{kj+1} - \overline{a^k} y^{kj} \qquad (\text{as } c^{kj} \neq 0) \quad (13)$$

We use the method of mathematical induction.

When $j = 1$, according to (10), (11) is equivalent to the $i-$condition (4).

Suppose that $\alpha^j, \lambda^j, \beta^j$ at any step $j > 1$ satisfy the sweep conditions of $i-$th condition concerning the solution of $k-$th equation of system (1), i.e., takes place the following:

$$\alpha^j y^{kj} + \lambda^j y^{kj+1} + \\ + \left[\sum_{s=k+1}^{L} (v_1^{is} y^{s1} + v_2^{is} y^{s2}) + \sum_{s=1}^{L} (w_1^{is} y^{sN_s-1} + w_2^{is} y^{sN_s}) \right] = \beta^j, \ j = 1,...,N_k . \tag{14}$$

Let's define the values for sweep coefficients $\alpha^{j+1}, \lambda^{j+1}, \beta^{j+1}$ for $(j+1)-$ th step:

$$\alpha^{j+1} y^{kj+1} + \lambda^{j+1} y^{kj+2} + \left[\sum_{s=k+1}^{L} (v_1^{is} y^{s1} + v_2^{is} y^{s2}) + \sum_{s=1}^{L} (w_1^{is} y^{sN_s-1} + w_2^{is} y^{sN_s}) \right] = \beta^{j+1} . \tag{15}$$

We take into account $k-$th equation of system (13) in (15):

$$\alpha^{j+1} y^{kj+1} + \lambda^{j+1} (\overline{d^{kj}} - \overline{b^{kj}} y^{kj+1} - \overline{a^{kj}} y^{kj}) + \\ + \left[\sum_{s=k+1}^{L} (v_1^{is} y^{s1} + v_2^{is} y^{s2}) + \sum_{s=1}^{L} (w_1^{is} y^{sN_s-1} + w_2^{is} y^{sN_s}) \right] = \beta^{j+1} .$$

From this equation we subtract equation (14), and obtain after grouping the following:
$[\alpha^{j+1} - \lambda^j - \lambda^{j+1} \overline{b^{kj}}] y^{kj+1} + [-\lambda^{j+1} \overline{a^{kj}} - \alpha^j] y^{kj} - [\beta^{j+1} - \lambda^{j+1} \overline{d^{kj}} - \beta^j] = 0$ .Given that this equality must hold for all possible solutions of $k-$th equation of system (1), we require from $\alpha^{j+1}, \lambda^{j+1}\ \beta^{j+1}$, satisfy the equality to zero of the expressions in square brackets. As a result, we obtain the necessary relations for sweep coefficients in the form (12). The theorem is proved.

Having completed the procedure for replacing the values of the $k-$th coefficient of the condition at y^{k1} and y^{k2} in the $i-$th condition with $y^{kN_{k-1}}$ and y^{kN_k} with the new values of the coefficients $\tilde{w}_1^{kN_k}$ and $\tilde{w}_2^{kN_k}$, we obtain a new condition equivalent to the $i-$th one. In this condition, there are no y^{k1} and y^{k2} values.

Next, we go to the following non-zero coefficients: v_1^{is}, v_2^{is}, $s > k$, until the conditions $v_1^{is} = 0$, $v_2^{is} = 0$, $s = 1,...,L$ are fulfilled. This means that the $i-$th condition has been fully transferred to the right. Then the whole procedure is performed for $(i+1)$-th term. If $(i+1) > M$, then all the conditions (4) are transferred to the right and the result are the conditions of the form (3.1) or (3.2) and equivalent to (4).

Conditions (6) and (7) represent a system of linear algebraic equations with respect to the vectors $y^{N-1}, y^N \in R^L$, after solving of which, from (1) the desired solution $y = \left(y^1,...,y^N\right)^*$ of the problem under consideration is determined.

Similarly to the above procedure of transferring conditions in the right end, a serial transfer is carried out in terms of the left end in order to obtain the conditions (8) or (9), equivalent to (4).

Let in the $i-$th condition among the vectors $w_1^{is}, w_2^{is}, s=1,...,L$, the first non-zero vector is $w_1^{ik} \neq 0$ or $w_2^{ik} \neq 0, 1 \leq k \leq L$, $w_1^{is}=0,\ w_2^{is}=0$, for $s<k$.

Definition 2. We say that the values α^j, λ^j and $\beta^j,\ j=1,...,N_k$ carry out the transfer of the $i-$th condition (4) with respect to the $k-$th unknown y^{kj} to the left, if for all vectors y^{kj}, satisfying the $k-$th equation of the system (1), we have the equalities

$$\alpha^j y^{kj} + \lambda^j y^{kj+1} + \left[\sum_{s=1}^{L}(v_1^{is} y^{s1} + v_2^{is} y^{s2}) + \sum_{s=k+1}^{L}(w_1^{is} y^{sN_s-1} + w_2^{is} y^{sN_s})\right] = \beta^j,\ j=1,...,N_k \qquad (15)$$

$$\alpha^{N_k} = w_1^{ik},\ \lambda^{N_k} = w_2^{ik},\quad \beta^{N_k} = R^i\ . \qquad (17)$$

It is clear that (15) with $j=N_k$ match the $i-$th condition (4).

If α^j, λ^j and $\beta^j,\ j=1,...,N_k$ are sweep coefficients, then from (15) at $j=1$ we get a new condition

$$\sum_{s=1}^{L}\tilde{v}_1^{is} y^{s1} + \sum_{s=1}^{L}\tilde{v}_2^{is} y^{s2} + \sum_{s=k+1}^{L} w_1^{is} y^{sN_s-1} + \sum_{s=k+1}^{L} w_2^{is} y^{sN_s} = \tilde{R}^i,\ i=1,...,M$$

equivalent to the $i-$th one, and where the following notations are introduced

$$\tilde{v}_1^{ik} = v_1^{ik} + \alpha^1,\ \tilde{v}_2^{ik} = v_2^{ik} + \lambda^1,\quad \tilde{v}_1^{is} = v_1^{is},\ \tilde{v}_2^{is} = v_2^{is},\quad s=1,...,L,\ s\neq k,\ \tilde{R}^i = \beta^1 .$$

This condition differs from the $i-$th condition (4) so that in its $i-$th part there are no terms with y^{kN_k-1}, y^{kN_k}. Further, this procedure is repeated until there is at least one coefficient w_1^{is}, w_2^{is}, different from zero. After that, if $i+1 \leq M$, the transfer of the next $(i+1)$ -th condition is carried out. The left sweep factors transferring $i-$th conditions to the left, can be determined from the following theorem.

Theorem 2. Suppose that the magnitudes α^j, λ^j and $\beta^j,\ j=1,...,N_k$ are defined by the following recurrent relations (discrete Cauchy problems):

$$\alpha^{j-1} = -\lambda^j \big/ c^{kj} \quad \lambda^{N_k} = w_2^{ik},\quad j=N_k-1, N_k-2,...,1,$$

$$\lambda^{j-1} = \alpha^j + \alpha^{j-1}\overline{b^{kj}},\ \alpha^{N_k} = w_1^{ik}, \qquad (18)$$

$$\beta^{j-1} = \alpha^{j-1}\overline{d^{kj}} - \beta^{j+1},\ \beta^{N_k} = R^i,\ j=N_k-1, N_k-2,...,1\ .$$

Then α^j, λ^j and β^j are left sweep coefficients for the $i-$th condition (4) with respect to the solution y^{kj} of the $k-$th equation of the system (1).

The proof is similar to the above proof of Theorem 1.

The very process of bringing all the conditions (4) and (5) to (8), (9) due to the transfer of values $y^{k N_k -1}$, $y^{k N_k}$ to the left end is similar to the process described above transfer conditions to the right. After the end of the transfer process we get a system of algebraic equations M for the vectors y^1, y^2. Solving this system, a recurrent calculation for required solutions y^{ks} of system (1) is carried out from left to right, $s=1,...,N_k$, $k=1,...,L$.

4. The results of numerical experiments

Consider the system of independent discrete equations consisting of five subsystems ($L=5,\quad N_s=201,\ s=1,...,5$):

$$160400 y^{1,i-1} - 320000 y^{1i} + 159600 y^{1,i+1} = -0{,}02 i - 2e^{0{,}005 i} + 2\ ,\ i=2,...,200,$$

$$159400 y^{2,i-1} - 320000 y^{2i} + 160600 y^{2,i+1} = -3{,}5 e^{0{,}0025 i} + 9 - \cos(0{,}005 i) - 3\sin(0{,}005 i)$$

$$320200 y^{3,i-1} - 640000 y^{3i} + 319800 y^{3,i+1} = -1, \tag{19}$$

$$160200 y^{4,i-1} - 320000 y^{4i} + 159800 y^{4,i+1} = 1 - 0{,}5 e^{0{,}0025 i} - ih\ ,$$

$$160200 y^{5,i-1} - 320000 y^{5i} + 159800 y^{5,i+1} = 0{,}01 i - 0{,}5 e^{0{,}0025 i} - 0{,}25\times 10^{-4} i^2$$

with the following ten unshared conditions, including the condition in the initial and final points:

$$y^{1,1} + y^{2,1} + y^{3,1} = 0\,, \tag{20}$$

$$y^{1,2} - y^{1,1} + y^{3,1} - y^{3,2} = 0\,, \tag{21}$$

$$y^{2,2} - y^{2,1} + y^{3,1} - y^{3,2} = 0\,, \tag{22}$$

$$y^{4,1} = 4\,, \tag{23}$$

$$y^{5,1} = -1\,, \tag{24}$$

$$y^{3,N_3} + y^{4,N_4} + y^{5,N_5} = 6\sqrt{e} - 1/6\,, \tag{25}$$

$$y^{3,N_3} - y^{3,N_3-1} - y^{5,N_5} + y^{5,N_5-1} = 0\,, \tag{26}$$

$$y^{4,N_4} - y^{4,N_4-1} - y^{5,N_5} + y^{5,N_5-1} = 0\,, \tag{27}$$

$$y^{1,N_1} = -1 + 2e\,, \tag{28}$$

$$y^{2,N_2} = 3 - 2\sqrt{e} + \cos(1)\,. \tag{29}$$

It is easy to verify that the solution of the problem (19) - (29) up 10^{-8} is a vector whose components are defined as follows

$$y^{1,i} = 0{,}25\times 10^{-4}(i-1)^2 + 2e^{0{,}005(i-1)} - 2,$$

$$y^{2,i} = 0{,}015(i-1) - 2e^{0{,}0025(i-1)} + \cos(0{,}005(i-1)),$$

$$y^{3,i} = 0{,}005(i-1) + 2e^{0{,}0025(i-1)} - 1, \tag{30}$$

$$y^{4,i} = 0{,}125\times 10^{-4}(i-1)^2 + 2e^{0{,}0025(i-1)} + 2,$$

$$y^{5,i} = 0{,}125\times 10^{-6}(i-1)^3 + 2e^{0{,}0025(i-1)} - 3\,.$$

Note that the system of equations (19) and (20) - (29) are obtained by simulation of finite difference approximations with partial differential equations of hyperbolic type, which describes the motion of the fluid in the pipe network.

System (19) determines the driving mode only in the first layer at discrete time points of the pipeline sections. All portions have equal lengths and divided into 200 portions.Conditions (20) and (25) define the law of the material balance at the nodal points of the network, the conditions (21) and (22), (26) and (27) - characterize the condition of continuity of the flow (equal pressure at the end portions adjacent to the node) The conditions (23) and (24), (28) and (29) determine the modes of external sources (the value of inflow and outflow of raw materials through source).

Given an equal number of non-zero coefficients at y^{s1} and $y^{s\,N_s}$ in the terms (20) - (29), the direction of transport conditions does't matter.

As a result of the transfer conditions (20) - (24) the right conditions were obtained in the form of an algebraic system (6), ten-dimentional matrices $\tilde{W}_1$ and $\tilde{W}_2$ which are presented in Tables 1 and 2, and the right side – vector $\tilde{R}$ had the form:

$\tilde{R}^* = [0.0035\ 9.7257\ 0.0\ \ 0.0\ 0.0050\ -0.0706\ 4.4366\ 0.2429\ 0.0324\ -0.0082]$.

Solving the resulting system of equations by Gauss with a choice of main member, was found the following vector

$y^N = (4.3998\ \ 4.4366\ \ 0.2403\ \ 0.2429\ \ 3.2771\ \ 3.2903\ \ 5.7886\ \ 5.8018\ \ 0.6203\ \ 0.6336)^*$.

Using this vector, recurrent calculations were carried out to find $y^{si}, \quad i = \overline{201,1}, \quad s = \overline{1,5}$ from the subsystems of the system (19). Accuracy of the results did not exceed

$$\max_s \max_i \left|\Delta y^{si}\right| \le 10^{-6} \ .$$

Table 1. Elements of matrice $\tilde{W}_1$ of system 3.1.

i \ j	1	2	3	4	5
1	-0.5501	-8.1103	-1	0	0
2	0.2231	0	-1	0	0
3	0	33.1173	-1	0	0
4	0	0	0	1	0
5	0	0	0	0	1
6	0	0	0	0	0
7	0	0	-1	0	1
8	0	0	0	-1	1
9	0	0	0	0	0
10	0	0	0	0	0

Table 2. Elements of matrice $\tilde{W}_2$ of system 3.1.

i \ j	1	2	3	4	5
1	0.5438	8.1040	0.9936	0	0
2	-0.2231	0	1	0	0
3	0	-33.1173	1	0	0
4	0	0	0	-0.9921	0
5	0	0	0	0	-0.9921
6	0	0	1	1	1
7	0	0	1	0	-1
8	0	0	0	1	-1
9	1	0	0	0	0
10	0	1	0	0	0

5. Conclusions

In this paper we consider the solution of systems of independent three-point difference equations of large dimension with "weak" and arbitrary connections between the individual equations, leading to indivisibility of defining the boundary conditions, and the matrix of the coupling conditions among the equations has sparse Jacobian. We have to solve such kind of systems repeatedly when optimizing the parameters of the objects of complex structure or when discretizing optimal control problems for processes described by ordinary or partial differential equations of second order. We have derived schemes and the corresponding formulas based on the idea of the conditions transfer method, which takes into account the characteristic features of the system’s Jacobi matrix.

Numerical results obtained in solving the problem on the calculation mode of unsteady flow of fluid in the pipe network loopback complex structure, to which is applied the implicit scheme of finite difference method.

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Ayrilmayan sərhəd şərtli üç nöqtəli diskret bir-birindən asili olmayan tənliklər sisteminin ədədi həlli

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XÜLASƏ

İşdə ayrılmayan sərhəd şərtlərinə malik üç nöqtəli diskret tənliklər sisteminin həllinə ədədi yanaşma təklif edilmişdir. Bu məsələnin həlli üçün düsturlar alınmış və təklif edilən üsulun tətbiqi üçün alqoritm verilmişdir. Məsələnin ədədi həllinin nəticələri verilmişdir ki, bu da təklif edilən üsulun effektivliyini göstərir.

Açar sözlər: diskret tənliklər sistemi, decompozisiya, ayrılmayan sərhəd şərtləri, sərhəd şərtinin köçürülməsi.

Численное решение системы независимых трехточечных дискретных уравнений с неразделенными граничными условиями

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РЕЗЮМЕ

В работе предложен численный подход к решению дискретных уравнений второго порядка с неразделенными краевыми условиями. Получены формулы для решения этой задачи и приводится алгоритм для применения предлагаемого метода. Приводятся результаты численного решения задачи, иллюстрирующие эффективность предлагаемого подхода.

Ключевые слова: системы дискретных уравнений, декомпозиция, неразделенные граничные условия, перенос краевых условий.