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A METHOD FOR SOLVING SINGULAR FUZZY MATRIX EQUATIONS

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Abstract. The fuzzy matrix equations $A\widetilde{x} = \widetilde{y}$, where A is a $n \times n$ singular crisp matrix is called singular fuzzy matrix equations. In this paper, a method for solving singular fuzzy matrix equations using Drazin inverse, is given.

Keywords: index of matrix, Drazin inverse, singular fuzzy linear system, matrix equations.

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1. Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh. System of simulations linear equations play major mathematics, physics, statistics. engineering and social sciences. One of the major applications using fuzzy number arithmetic is treating linear systems their parameters are all or partially represented by fuzzy numbers [3].

A $n \times m$ linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number vector, is called fuzzy linear system. Friedman et al. [7] introduced a general model for solving fuzzy linear system.

Solving fuzzy linear system is a current issue in recent years [1, 2, 3]. In [4] the original fuzzy linear system with the nonsingular matrix A is replaced by two $n \times n$ crisp linear system. On the inconsistent fuzzy matrix equations

$$A\widetilde{x} = \widetilde{y}$$

and its fuzzy least-squares solutions is discussed in [10].

Index of matrix and Drazin inverse for any $n \times n$ matrix, even singular matrices, exists and are unique. Index of matrix and Drazin inverse in solving consistent or inconsistent singular linear system, are used [5, 11].

The effect of index of matrix in solving singular fuzzy linear system, is explained [9]. A fuzzy matrix equations whose coefficient matrix is singular crisp matrix, is called singular fuzzy matrix equations. In this paper, a method for solving consistent or inconsistent singular fuzzy matrix equations, is proposed.

The rest of this paper is organized as follows, section 2 gives a Definitions and Basic results. In section 3, some new results on the singular matrices and singular fuzzy matrix equations, is given. The effect of Drazin inverse in solving singular fuzzy matrix equations, is investigated, in section 4. In section 5, two numerical example gives to show the usefulness of the proposed method. Section 6 ends the paper with the conclusion remarks.

2. Preliminaries

In this section, the following Definitions and Basic results, is are given.

Definition 1. The index of matrix $A \in C^{n\times n}$ is the dimension of largest Jordan block corresponding to the zero eigenvalue of A and is denoted by $ind\ (A)$.

Definition 2. Let $A \in C^{n\times n}$, with $ind(A) = k$. The matrix X of order n is the Drazin inverse of A, denoted by A^D, if X satisfies the following conditions

$$AX = XA, \quad XAX = X, \quad A^k XA = A^k. \tag{1}$$

When $ind(A) = 1$, A^D is called the group inverse of A, and is denoted by A_g.

Theorem 1. [11] Let $A \in C^{n\times n}$. with $ind(A) = k$, $rank(A^k) = r$. We may assume that the Jordan normal form of A has the form as follows

$$A = P\begin{pmatrix} D & 0 \\ 0 & N \end{pmatrix}P^{-1},$$

where P is a nonsingular matrix, D is a nonsingular matrix of order r, and N is a nilpotent matrix of index k. Then, we can write the Drazin inverse of A in the form

$$A^D = P\begin{pmatrix} D^{-1} & 0 \\ 0 & 0 \end{pmatrix}P^{-1}.$$

Theorem 2. [4] $A^D b$ is a solution of

$$Ax = b, \text{ where } k = ind(A) \tag{2}$$

if and only if $b \in R(A^k)$, and $A^D b$ is an unique solution of (2) provided that $x \in R(A^k)$.

Definition 3. A fuzzy number $\tilde{u}$ in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0 \le r \le 1$, which satisfy the following requirements

1. $\underline{u}(r)$ is a bounded left continuous non-decreasing function over $[0,1]$.
2. $\overline{u}(r)$ is a bounded left continuous non-increasing function over $[0,1]$.
3. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

The set of all these fuzzy numbers is denoted by E.

Definition 4. For arbitrary fuzzy numbers $\tilde{x} = (\underline{x}(r), \overline{x}(r))$, $\tilde{y} = (\underline{y}(r), \overline{y}(r))$ and $k \in R$, we may define the addition and the scalar multiplication of fuzzy numbers by using the extension principle as [9]

1. $\tilde{x} + \tilde{y} = (\underline{x}(r) + \underline{y}(r), \overline{y}(r) + \overline{y}(r)),$
2. $k\tilde{x} = \begin{cases} (k\underline{x}, k\overline{x}), & k \ge 0 \\ (k\overline{x}, k\underline{x}), & k < 0. \end{cases}$

Definition 5. The matrix system

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \cdots & \tilde{x}_{nn} \end{bmatrix} = \begin{bmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{y}_{n1} & \cdots & \tilde{y}_{nn} \end{bmatrix}, \tag{3}$$

where $A=(a_{ij})$ is a real singular matrix, and the elements $\tilde{b}_{ij}$ in the right-hand side matrix are fuzzy numbers is called singular fuzzy matrix equations. A singular fuzzy matrix equations (3) can be extended into a crisp matrix equation as follows

$$\begin{bmatrix} s_{11} & \cdots & s_{1,2n} \\ \vdots & \ddots & \vdots \\ s_{2n,1} & \cdots & s_{2n,2n} \end{bmatrix} \begin{bmatrix} \underline{x}_{11} & \cdots & \underline{x}_{1n} \\ \vdots & \ddots & \vdots \\ \underline{x}_{n1} & \cdots & \underline{x}_{nn} \\ -\overline{x}_{11} & \cdots & -\overline{x}_{n1} \\ \vdots & \ddots & \vdots \\ -\overline{x}_{n1} & \cdots & -\overline{x}_{nn} \end{bmatrix} = \begin{bmatrix} \underline{y}_{11} & \cdots & \underline{y}_{1n} \\ \vdots & \ddots & \vdots \\ \underline{y}_{n1} & \cdots & \underline{y}_{nn} \\ -\overline{y}_{11} & \cdots & -\overline{y}_{n1} \\ \vdots & \ddots & \vdots \\ -\overline{y}_{n1} & \cdots & -\overline{y}_{nn} \end{bmatrix},$$

where s_{ij} are determined as follows:

$$a_{ij} \geq 0 \quad \Rightarrow \quad s_{ij} = a_{ij}\ , \quad s_{i+n,j+n} = a_{ij}$$
$$a_{ij} < 0 \quad \Rightarrow \quad s_{ij+n} = -a_{ij}\ , \quad s_{i+n,j} = -a_{ij}$$

while all the remaining s_{ij} are taken zero. Using matrix notation we get

$$SX = Y, \tag{4}$$

where $S=(s_{ij})\geq 0$, $1 \leq i \leq 2n$, $1 \leq j \leq 2n$ and

$$x_j = \begin{bmatrix} \underline{x}_j \\ -\overline{x}_j \end{bmatrix} = \begin{bmatrix} \underline{x}_1 \\ \vdots \\ \underline{x}_n \\ -\overline{x}_1 \\ \vdots \\ -\overline{x}_n \end{bmatrix}, 1 \leq j \leq n, \quad X = (x_1, x_2, \cdots, x_n), \tag{5}$$

$$y_j = \begin{bmatrix} \underline{y}_j \\ -\overline{y}_j \end{bmatrix} = \begin{bmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_n \\ -\overline{y}_1 \\ \vdots \\ -\overline{y}_n \end{bmatrix}, 1 \leq j \leq n, \quad Y = (y_1, y_2, \cdots, y_n). \tag{6}$$

The structure of S implies that $S=(s_{ij})\geq 0$, $1 \leq i \leq 2n$, $1 \leq j \leq 2n$ and that

$$S = \begin{bmatrix} B & C \\ C & B \end{bmatrix},$$

where B contains the positive entries of A and C contains the absolute value of the negative entries of A, i.e., $A = B - C$.

Theorem 3. [10] Let $A\tilde{x} = \tilde{y}$ be a fuzzy linear system of equations and $SX = Y$ be extended system of it. The system $A\tilde{x} = \tilde{y}$ is a consistent fuzzy linear system, if and only if

$$rank\,[S] = rank\,[S|Y],$$

Definition 6. [1] Let $X = \{(\underline{x}_i(r), -\overline{x}_i(r))\,;1 \le i \le n\}$ be a solution of (4). The fuzzy number vector $U = \{(\underline{u}_i(r), \overline{u}_i(r))\,;1 \le i \le n\}$ defined by

$$\underline{u}_i(r) = \min\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1)\},$$
$$\overline{u}_i(r) = \max\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1)\}$$

is called a fuzzy solution of (4). If $(\underline{x}_i(r), \overline{x}_i(r))\,;1 \le i \le n$, are all fuzzy numbers and $\underline{x}_i(r) = \underline{u}_i$, $\overline{x}_i(r) = \overline{u}_i(r)\,;1 \le i \le n$, then U is called a strong fuzzy solution. Otherwise, U is a weak fuzzy solution.

3. New results on the singular matrices

In this section, some new results on the Drazin inverse and index of matrix, are given.

Theorem 4. Let A be a singular matrix. Then matrix

$$\begin{bmatrix} B & C \\ C & B \end{bmatrix}$$

is a singular matrix, wherein $A = B - C$.

Proof. The same as the proof of Theorem 1 in [7], by elementary row operations we obtain

$$\begin{bmatrix} B & C \\ C & B \end{bmatrix} = \begin{bmatrix} B+C & C+B \\ C & B \end{bmatrix} = \begin{bmatrix} B+C & 0 \\ C & B-C \end{bmatrix}.$$

Clearly, $|S| = |B+C||B-C| = |B+C||A|$. A is a singular matrix. Therefore $|S| = 0$.

Theorem 5. Let $A\tilde{x} = \tilde{y}$ be a consistent singular fuzzy matrix equations. The extended system of it, has a set of solutions.

Proof. The system $SX = Y$ is consistent. i.e. $rank\,[S] = rank\,[S|Y]$. The matrix equations $SX = Y$ by (5) and (6) is equivalent to the following linear equations

$$Sx_j = y_j, \quad j = 1,2,\cdots,n.$$

Since,

$$rank\ [S] \le rank\ [S|y_j(r)] \le rank\ [S|Y]$$

we have $rank\ [S] \le rank\ [S|y_j(r)]$ for all $j = 1,2,\cdots,n$. By theorem2.8 we know all linear equations $Sx_j = y_j$, $j = 1,2,\cdots,n$ have solutions, i.e. the matrix equations $SX = Y$ has solution. By theorem 4 S is a singular matrix. Therefore from [8] $SX = Y$ has a set of solutions.

Corollary 1. The extended system of an inconsistent singular fuzzy matrix equations, has a least-squares solutions.

Corollary 2. Let $SX = Y$ be the extended singular matrix equations of the inconsistent singular fuzzy matrix equations $A\tilde{x} = \tilde{y}$. In order to obtain the Drazin inverse for solves consistent or inconsistent singular linear system $SX = Y$ where $ind(S) = k$ through solving the consistent singular linear system

$$S^k SX = S^k Y \ . \qquad (7)$$

Corollary 3. Let $ind\left(\begin{bmatrix} B & C \\ C & B \end{bmatrix}\right) = 1$, by

$$\begin{bmatrix} B & C \\ C & B \end{bmatrix}\begin{bmatrix} B & C \\ C & B \end{bmatrix} = \begin{bmatrix} BB+CC & BC+CB \\ BC+CB & BB+CC \end{bmatrix}.$$

The structure of system (7) is preserved. If $ind(S) \ne 1$ the system (7) may not preserve its structure.

Theorem 6. Let A be a symmetric matrix, show that

$$\begin{bmatrix} B & C \\ C & B \end{bmatrix}$$

is a symmetric matrix, wherein $A = B - C$.

Proof. From [6], we have

$$A^T = (B-C)^T = B^T - C^T = B - C.$$

Therefore $B^T = B, C^T = C$. We know that

$$\begin{bmatrix} B & C \\ C & B \end{bmatrix}^T = \begin{bmatrix} B^T & C^T \\ C^T & B^T \end{bmatrix} = \begin{bmatrix} B & C \\ C & B \end{bmatrix}.$$

4. A method for solving singular fuzzy matrix equations

In this section, A new method using Drazin inverse for solving singular fuzzy matrix equations, is proposed.

Theorem 7. $S^D Y$ is a solution of

$$SX = Y \text{, where } k = ind(S)$$

if and only if $y_j \in R(S^k), \quad j = 1,2,\cdots,n$.

Proof. See [4].

Proposed method. By noting the structure of S and Equations (5) and (6) for any column of matrix X and Y, for $1 \le j \le n$ we obtain the following linear system

$$\begin{cases} B(\underline{x}_j) + C(-\overline{x}_j) = \underline{y}_j \\ C(\underline{x}_j) + B(-\overline{x}_j) = -\overline{y}_j. \end{cases}$$

Which is a crisp function linear system. If (8) be consistent, by adding and then subtracting the part this equation, we obtain

$$\begin{cases} (B+C)(\overline{x}_j - \underline{x}_j) = \overline{y}_j - \underline{y}_j \\ (B-C)(\underline{x}_j + \overline{x}_j) = \overline{y}_j + y_j. \end{cases}$$

We can get

$$\begin{cases} E(\overline{x}_j - \underline{x}_j) = \overline{y}_j - \underline{y}_j \\ A(\underline{x}_j + \overline{x}_j) = \overline{y}_j + y_j, \end{cases}$$

wherein , $E = B + C$ and/or

$$\begin{cases} E\sigma_j = \overline{y}_j - \underline{y}_j \\ A\delta_j = \overline{y}_j + y_j \end{cases},$$

where $\sigma_j = \overline{x}_j - \underline{x}_j$, $\delta_j = \overline{x}_j + \underline{x}_j$. By adding and subtracting the two solutions of above systems we obtain

$$\overline{x}_j = \frac{\sigma_j + \delta_j}{2}, \quad \underline{x}_j = \frac{\delta_j - \sigma_j}{2}.$$

Corollary 4. In order to obtain to obtain the Drazin inverse for solving inconsistent singular linear system

$$SX = Y, \text{ where } k = ind(S)$$

and by corollary 2, we can get

$$X = (S^k S)^D S^k Y$$

Corollary 5. Let

$$SX = Y, \text{ where } 1 < ind(S)$$

be a inconsistent singular linear system. In this case, by corollary 2 we perform proposed method, on the singular consistent linear system

$$S^k SX = S^k Y \text{ where } k = ind(S).$$

5. Numerical examples

We now give the following examples to explain the present results.

Example 1. Consider the following singular fuzzy matrix equations

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} \\ \tilde{x}_{21} & \tilde{x}_{22} \end{bmatrix} = \begin{bmatrix} (-2,-1-r) & (r,2-r) \\ (2+r,3) & (4+2r,8-2r) \end{bmatrix} \tag{8}$$

The extended 4×4 matrix equations is

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} \underline{x}_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \underline{x}_{22} \\ -\overline{x}_{11} & -\overline{x}_{12} \\ -\overline{x}_{21} & -\overline{x}_{22} \end{bmatrix} = \begin{bmatrix} -2 & r \\ 2+r & 4+2r \\ r+1 & r-2 \\ -3 & 2r-8 \end{bmatrix} \tag{9}$$

Since $rank\,[S] = rank\,[S|Y]$, then the system (8) is inconsistent. The index of (9) equals two, then

$$S^2SX = S^2Y. \tag{10}$$

We have

$$S^3 = P^{-1}\begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}P, \quad P = \begin{bmatrix} -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} & \frac{11}{4} & -\frac{13}{4} \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

and by theorem 1 , we get

$$(S^3)^D = P^{-1}\begin{bmatrix} \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}P = \begin{bmatrix} \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \end{bmatrix}.$$

By Corollary 4 we have $X = (S^2S)^D(S^2Y)$. Therefore,

$$\begin{bmatrix} \underline{x}_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \underline{x}_{22} \\ -\overline{x}_{11} & -\overline{x}_{12} \\ -\overline{x}_{21} & -\overline{x}_{22} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}+\frac{1}{4}r & -\frac{3}{4}+\frac{3}{4}r \\ -\frac{1}{4}+\frac{1}{4}r & -\frac{3}{4}+\frac{3}{4}r \\ -\frac{1}{4}+\frac{1}{4}r & -\frac{3}{4}+\frac{3}{4}r \\ -\frac{1}{4}+\frac{1}{4}r & -\frac{3}{4}+\frac{3}{4}r \end{bmatrix}$$

is a strong fuzzy solution of (10). The index of coefficient matrix of system (9) equal two. Therefore we perform proposed method, on the system (10).

Example 2. Consider the following consistent singular fuzzy matrix equations

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} \\ \tilde{x}_{21} & \tilde{x}_{22} \end{bmatrix} = \begin{bmatrix} (-2,-1-r) & (r,2-r) \\ (-4,-2-2r) & (2r,4-2r) \end{bmatrix}. \tag{11}$$

The extended 4×4 matrix equations is

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \underline{x}_{22} \\ -\overline{x}_{11} & -\overline{x}_{12} \\ -\overline{x}_{21} & -\overline{x}_{22} \end{bmatrix} = \begin{bmatrix} -2 & r \\ -4 & 2r \\ r+1 & r-2 \\ 2r+2 & 2r-4 \end{bmatrix}. \tag{12}$$

The system (11) is consistent. By theorem 2.3 we have

$$S = P^{-1} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} P, \quad P = \begin{bmatrix} -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & -\frac{1}{3} & \frac{8}{3} & -\frac{4}{3} \\ 0 & 0 & 2 & -1 \end{bmatrix}.$$

Then, the Drazin inverse of matrix S is

$$S = P^{-1} \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} P, \quad P = \begin{bmatrix} \frac{5}{9} & -\frac{4}{9} & -\frac{4}{9} & \frac{5}{9} \\ \frac{10}{9} & -\frac{8}{9} & -\frac{8}{9} & \frac{10}{9} \\ -\frac{4}{9} & \frac{5}{9} & \frac{5}{9} & -\frac{4}{9} \\ -\frac{8}{9} & \frac{10}{9} & \frac{10}{9} & -\frac{8}{9} \end{bmatrix}.$$

By theorem 7 we have

$$\begin{bmatrix} \underline{x}_{11} & \underline{x}_{12} \\ \underline{x}_{21} & \underline{x}_{22} \\ -\overline{x}_{11} & -\overline{x}_{12} \\ -\overline{x}_{21} & -\overline{x}_{22} \end{bmatrix} = \begin{bmatrix} \frac{4}{3}+\frac{2}{3}r & -\frac{4}{3}+\frac{1}{3}r \\ \frac{8}{3}+\frac{4}{3}r & -\frac{8}{3}+\frac{2}{3}r \\ -\frac{5}{3}-\frac{1}{3}r & \frac{2}{3}+\frac{1}{3}r \\ -\frac{10}{3}-\frac{2}{3}r & \frac{4}{3}+\frac{2}{3}r \end{bmatrix}.$$

Obviously x_{11}, x_{21} are not fuzzy number, and hence we can obtain the weak fuzzy solution as follows

$$U = \begin{bmatrix} (\frac{4}{3}+\frac{2}{3}r,2) & (-\frac{4}{3}+\frac{1}{3}r,-\frac{2}{3}-\frac{1}{3}r) \\ (\frac{8}{3}+\frac{4}{3}r,4) & (-\frac{8}{3}+\frac{2}{3}r,-\frac{4}{3}-\frac{2}{3}r) \end{bmatrix}.$$

The system (12) is a consistent singular linear system with index one. Therefore, we have

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_{11} \\ \underline{x}_{21} \\ -\overline{x}_{11} \\ -\overline{x}_{21} \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ r+1 \\ 2r+2 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_{12} \\ \underline{x}_{22} \\ -\overline{x}_{12} \\ -\overline{x}_{22} \end{bmatrix} = \begin{bmatrix} r \\ 2r \\ r-2 \\ 2r-4 \end{bmatrix}.$$

By proposed method we have,

$$\begin{cases} E\sigma_1 = \overline{y}_1 - \underline{y}_1 \\ A\delta_1 = \overline{y}_1 + y_1 \end{cases}$$

and

$$\begin{cases} E\sigma_2 = \overline{y}_2 - \underline{y}_2 \\ A\delta_2 = \overline{y}_2 + y_2 \end{cases}.$$

By solving these systems, we can get

$$\sigma_1 = E^D(\overline{y}_1 - \underline{y}_1) = \begin{bmatrix} -\frac{1}{3}r + \frac{1}{3} \\ -\frac{2}{3}r + \frac{2}{3} \end{bmatrix}, \quad \delta_1 = A^D(\overline{y}_1 + \underline{y}_1) = \begin{bmatrix} r+3 \\ 2r+6 \end{bmatrix}$$

and

$$\sigma_2 = E^D(\overline{y}_2 - \underline{y}_2) = \begin{bmatrix} -\frac{2}{3}r + \frac{2}{3} \\ -\frac{4}{3}r + \frac{4}{3} \end{bmatrix}, \quad \delta_1 = A^D(\overline{y}_1 + \underline{y}_1) = \begin{bmatrix} -2 \\ -4 \end{bmatrix}.$$

Finally, the solution of consistent system(12) by proposed method is

$$\begin{bmatrix} \frac{4}{3} + \frac{2}{3}r & -\frac{4}{3} + \frac{1}{3}r \\ \frac{8}{3} + \frac{4}{3}r & -\frac{8}{3} + \frac{2}{3}r \\ -\frac{5}{3} - \frac{1}{3}r & \frac{2}{3} + \frac{1}{3}r \\ -\frac{10}{3} - \frac{2}{3}r & \frac{4}{3} + \frac{2}{3}r \end{bmatrix}.$$

6. Conclusions

The original fuzzy linear system $A\tilde{x} = \tilde{y}$ with a crisp matrix $A = B - C$ is replaced by two $n \times n$ crisp linear systems $(B - C)(\underline{x} + \overline{x}) = (\underline{y} + \overline{y})$ and $(B + C)(\overline{x} - \underline{x}) = (\overline{y} - \underline{y})$. Using ordinary inverse $(\underline{x} + \overline{x}) = (B - C)^{-1}(\underline{y} + \overline{y})$ and $(\overline{x} - \underline{x}) = (B + C)^{-1}(\overline{y} - \underline{y})$. If and only if (B − C) and (B + C) are both crisp nonsingular matrices[4]. In this paper, Asady's method is extended and on the consistent singular fuzzy matrix equations, is performed. The consistent singular fuzzy matrix equations $A\tilde{x} = \tilde{y}$ with a crisp matrix $A = B - C$ is replaced by two $n \times n$ singular crisp linear systems $(B - C)(\underline{x} + \overline{x}) = (\underline{y} + \overline{y})$ and $(B + C)(\overline{x} - \underline{x}) = (\overline{y} - \underline{y})$. Using Drazin inverse by theorem 2.4 we have $(\underline{x} + \overline{x}) = (B - C)^{D}(\underline{y} + \overline{y})$ and $(\overline{x} - \underline{x}) = (B + C)^{D}(\overline{y} - \underline{y})$ wherein $(B - C)$ and $(B + C)$ are both crisp singular matrices.

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Qeyri-səlis sinqulyar matris tənliyin həll üsulu

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XÜLASƏ

Qeyri-səlis matris $A\tilde{x} = \tilde{y}$ tənliyi, A-sinqulyar qeyri-səlis $n \times n$ ölçülü matris olduqda sinqulyar qeyri-səlis matris tənliyi adlanır. Məqalədə Drezin tərsi anlayışından istifadə edərək qeyri-səlis sinqulyar matris tənliyin həlli üsulu verilir.

Açar sözlər: matrisin indeksi, Drazin tərsi, qeyri-səlis sinqulyar xətti sistem, matris tənlik.

Метод решения нечеткого сингулярного матричного уравнения

М.М. Никуе

РЕЗЮМЕ

Нечеткое матричное уравнение $A\tilde{x} = \tilde{y}$, где A-сингулярная четкая матрица размерности $n \times n$, называется сингулярным нечетким матричным уравнением. В работе используя Дразин обратное дается метод для решения сингулярного матричного нечеткого уравнения.

Ключевые слова: индекс матрицы, Дразин обратное, сингулярная нечеткая линейная система, матричное уравнение.