TWMS J. Pure Appl. Math., V.13, N.2, 2022, pp.211-221

CONSTRUCTING AN OPTIMAL CONTROLLER FOR MANEUVER OF QUADROTOR IN 3-D SPACE*

F.A. ALIEV[1,2], M.M. MUTALLIMOV[1,2], $\boxed{\text{A.A. TUNIK}^3}$, N.I. VELIEVA[1,4], U.Z. RASULOVA[5]
S.M. MIRSAABOV[1]

Abstract. In this paper, based on a linearized mathematical model of the dynamics of a quadrotor, an optimal controller is constructed to control the spatial motion of a quadrotor when it is raising to a given altitude with simultaneous heading maneuver in a horizontal plane. Moreover, the maneuver trajectory must satisfy certain boundary conditions. The feature of this paper is the application of LQR synthesis to optimize the control of the translational motions of a quadrotor along all three axes of the inertial coordinate system, as well as to control its rotational motion relative to the vertical axis. The effectiveness of the developed control laws has been proven by modeling the above-described spatial motion of a quadrotor using the MATLAB package.

Keywords: quadrotor, dynamic model, optimal controller, LQR-synthesis given altitude, yaw angle.

AMS Subject Classification: 37N10, 37N20, 37N40.

1. Introduction

The development of computer technology and information technologies currently allow the use of mathematical modeling as a tool for solving control problems of complex technical systems at a qualitatively new level. The application of the modern mathematical packages of applying programs makes it possible to carry out multilateral studies with high accuracy and with minimal expenditure of all resources. One of such complex systems is unmanned aerial vehicles (UAVs), in particular quadrotors, which have recently become increasingly popular as affordable and relatively inexpensive technical means for distant collecting information, monitoring the environment, delivering small-sized cargo, and several other problems.

Currently, in engineering practice, a significant place is occupied by mathematical problems, which, in one setting or another, can be associated with the control of mechanical systems [1]. It should be noted that among the various problems of stability and motion control of the UAV, the problem of developing mathematical modeling and motion control of the quadrotor is relevant. [3, 6].

A quadrotor is a very maneuverable aircraft that has low dissipation since its dynamics is highly susceptible to atmospheric disturbances due to its relatively small mass and relatively

*Author F.A. was partially supported by the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No.873071.

[1]Institute of Applied Mathematics, Baku State University, Baku, Azerbaijan
[2]Institute of Information Technologies of ANAS, Baku, Azerbaijan
[3]National Aviation University, Kiev, Ukraine
[4]Azerbaijan Technical University, Baku, Azerbaijan
[5]Azerbaijan University of Architecture and Construction, Baku, Azerbaijan
e-mail: f_aliev@yahoo.com, mmutallimov@bsu.edu.az

Manuscript received March 2021.

low speed. Its control system should solve the problems of angular and spatial stabilization, the quadrotor reaching a given altitude (takeoff), landing, hovering, and flight along a given trajectory [5, 10]. In the general case, rather high requirements are imposed on the quadrotor control system in terms of accuracy and speed, taking into account the specified restrictions.

There are a large number of works devoted to modeling the quadrotors motion [2-10], where various versions of the equations of motion with various systems of automatic control and stabilization are given.

Thus, in [3], a dynamic model of a quadrotor was constructed, which was obtained using the Lagrange method. In [6], based on a mathematical model of the apparatus motion, algorithms for the synthesis of quadrotor control are presented. Here, two options for controlling the horizontal motion of the apparatus are considered. The first one considers the quadrotor position control, while the second one considers the control of the quadrotor cruising speed.

This work proposes an algorithm for constructing a controller for performing the maneuver in the $3-D$ space, consisting of the quadrotor climb to a given altitude along the axis z, and turning in the horizontal plane at the given heading angle ψ_d when the aircraft is moving. At the same time, we impose the following boundary conditions on the trajectory of this maneuver. Let us have the arbitrary starting point of the quadrotor on the horizontal plane of the fixed inertial frame $P_s = [x_0, y_0, 0]^T$ and the arbitrary attitude vector $A_i = [\phi_0, \theta_0, \psi_0]^T$. Here ϕ, θ are the roll and the pitch angles respectively. Then the final position of the quadrotor must have the form $P_f = [0, 0, z_d]^T$, where z_d is desired altitude, and the final value of attitude vector must be $A_f = [0, 0, \psi_d]^T$.

Based on the proposed control algorithm, the calculations, which provide the simulation of the closed-loop system, were carried out, and the specific examples illustrate their results.

2. Problem statement

Consider a quadrotor Fig.1 with given physical parameters, the motion of which can be controlled by changing the rotation speed of the propellers.

Figure 1. Quadrotor.

The aircraft moves relatively to a fixed inertial frame of reference, connected with the Earth and given by coordinate axes Ox, Oy, and Oz perpendicular to each other, and the Oz axis is directed opposite to the gravity vector. As it was stated above, the problem is to make the quadrotor move from the starting point (x_0, y_0, 0), with initial angles (θ_0, φ_0, ψ_0) to a given point (0, 0, z_d) with angles (0, 0, ψ_d), which means raising the quadrotor to a given altitude and turning along the yaw angle.

The further statement of the problem is justified by the fact that, due to the symmetry of the construction of the quadrotor concerning its axes, it is possible to decompose the control system for its spatial motion into four independent subsystems. These include systems controlling motions along the axes Ox, Oy, Oz, and rotational motion relative to the vertical axis Oz [3, 9]. Experimental verification of this assumption showed that the errors of such an approximation are quite small, and for practical application, they can be neglected [3, 9].

Fig.2 represents the kinematic scheme of the quadrotor. As noted in [3], let $\xi = [x \;\; y \;\; z]'$ be the radius vector of the quadrotor center, ψ, θ, φ are the yaw, pitch, and roll angles, respectively. f_i is the lifting force of the i-th propeller with a motor $M_i (i = \overline{1,4})$. The prime here and below denotes transposition.

Let

$$u = f_1 + f_2 + f_3 + f_4,$$

total thrust, where $f_i = k_i \omega_i^2$, $(i = \overline{1,4})$, $k_i > 0$ are experimentally determining thrust coefficients, and ω_i is the angular velocity of i-th motor $M_i (i = \overline{1,4})$. In addition, suppose that the rotational torques are defined as

$$\tilde{\tau}_\psi = [(f_2 + f_4) - (f_1 + f_3)] \cdot l \, ,$$

$$\tilde{\tau}_\theta = (f_2 - f_4) \cdot l,$$

$$\tilde{\tau}_\varphi = (f_3 - f_1) \cdot l,$$

where l is the distance from the motor to the center of gravity of the quadrotor. These rotation torques are the functions of the squares of the motors' rotation speeds.

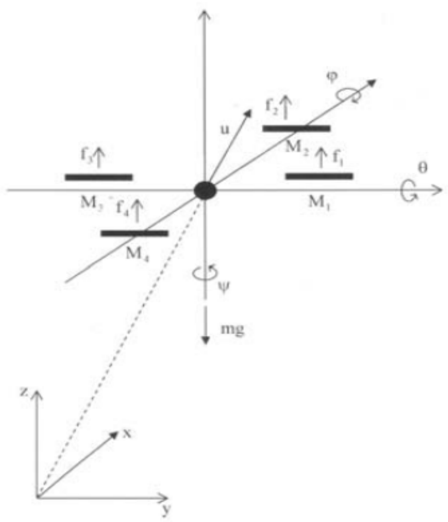


Figure 2. Kinematic scheme of a quadrotor.

According to [3, 6], the equations of motion of such system have the form:

$$m\ddot{x} = -u \sin\theta, \tag{1}$$

$$m\ddot{y} = u \cos\theta \sin\varphi, \tag{2}$$

$$m\ddot{z} = u\cos\theta\cos\varphi - mg, \tag{3}$$

$$\ddot{\psi} = \tilde{\tau}_\psi, \tag{4}$$

$$\ddot{\theta} = \tilde{\tau}_\theta, \tag{5}$$

$$\ddot{\phi} = \tilde{\tau}_\varphi. \tag{6}$$

In the equations (1) – (6), m – is the mass of the quadrotor, $g = 9,8/$–is the gravity acceleration, u along with $\tilde{\tau}_\psi,\ \tilde{\tau}_\theta,\ \tilde{\tau}_\varphi$ – are control actions that are functions f_i. Note that the action $u$is used to control the altitude of the vehicle, the control$\tilde{\tau}_\psi$ allows the heading angle stabilization.
Now, under the assumption$\cos\theta\cos\varphi \neq 0$, the control of the flight altitude u of the apparatus we determine by the following relationship:

$$u = (r_1 + mg)\,\frac{1}{\cos\theta\cos\varphi}. \tag{7}$$

As r_1, in contrast to [3, 6], we take

$$r_1 = a_1\dot{z} + a_2 z + a_3. \tag{8}$$

Then, taking into account (7) and (8), we obtain from (3)

$$m\ddot{z} = a_1\dot{z} + a_2 z + a_3.$$

Similarly, to control the yaw angle, we take

$$\tilde{\tau}_\psi = b_1\dot{\psi} + b_2\psi + b_3. \tag{9}$$

Then at $\cos\theta\cos\varphi \neq 0$ and taking into account (7)–(9), from formulas (3), (4) we obtain:

$$\ddot{z} = \frac{1}{m}(a_1\dot{z} + a_2 z + a_3), \tag{10}$$

$$\ddot{\psi} = b_1\dot{\psi} + b_2\psi + b_3. \tag{11}$$

In this case, equations (1), (2) pass to the following equations

$$m\ddot{x} = -\,(r_1 + mg)\,\frac{\tan\theta}{\cos\varphi}, \tag{12}$$

$$m\ddot{y} = (r_1 + mg)\tan\varphi, \tag{13}$$

and equations (5), (6) remain unchanged. Note that the unknown coefficients a_1, a_2, $a_3 b_1$, b_2, b_3 must be chosen from the condition of asymptotic stability in the vertical direction and in the yaw angle, which, in turn, will ensure that the condition $z \to z_d,\ \psi \to \psi_d$ is satisfied, for given z_d, ψ_d.
It is known from the theory of differential equations that the roots of the characteristic equation

$$k^2 - \frac{a_1}{m}k - \frac{a_2}{m} = 0,$$

for differential equation (10) are located in the left complex half-plane if a_1, a_2 are negative, i.e. $Re(k_i) < 0$, $i = 1, 2$. Since $z = -\frac{a_3}{a_2}$ is the particular solution of the inhomogeneous equation (10), the general solution of this equation will have the form

$$z = c_1 e^{k_1 t} + c_2 e^{k_2 t} - \frac{a_3}{a_2},$$

for which $z(t) \to -\frac{a_3}{a_2}$ at $t \to \infty$. Using notations$a_1 = -a_{z_1}$, $a_2 = -a_{z_2}$, $-\frac{a_3}{a_2} = z_d$, and expression (8), we obtain the following formula for r_1:

$$r_1 = a_1 \dot{z} + a_2 z + a_3 = -a_{z_1} \dot{z} - a_{z_1} z + a_{z_1} z_d = -a_{z_1} \dot{z} - a_{z_1} (z - z_d), \tag{14}$$

which corresponds to formula (22) from [6]. Then equation (10) transforms into the equation

$$\ddot{z} = \frac{1}{m} \left(-a_{z_1} \dot{z} - a_{z_2} (z - z_d) \right). \tag{15}$$

Similarly, from equation (11) we obtain

$$\ddot{\psi} = -a_{\psi_1} \dot{\psi} - a_{\psi_2} (\psi - \psi_d). \tag{16}$$

Note that for positive values of the coefficients a_{ψ_1}, a_{ψ_2}, a_{z_1}, a_{z_2}, in (15), (16) the condition of asymptotic stability of these systems is satisfied, which, in turn, ensures the fulfillment of $\psi \to \psi_d, \quad z \to z_d$.

It should be noted here that we will obtain an identical result by applying the vertical motion stabilization algorithm.

Let

$$p_1 = [(z - z_d), \ \frac{d(z - z_d)}{dt}]'.$$

Let denote

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ u_1 = \frac{r_1}{m}.$$

Then from (14), (15), we obtain

$$\begin{aligned} \dot{p}_1 &= A_1 p_1 + B_1 u_1, \\ p_1(0) &= p_1^0. \end{aligned} \tag{17}$$

Further, in this case, we will use the following integral-quadratic performance index

$$J_1 = \int_0^\infty \left(p_1' Q_1 p_1 + u_1' R_1 u_1 \right) dt,$$

where the matrices $Q_1 R_1$ are providing solutions to the synthesis problem for the translational motion control along the axis z. As a result of solving the synthesis problem and applying the standard LQR synthesis procedure, we obtain the state feedback coefficients $u_1 = -K_z p_1$ in the form $K_z = \frac{1}{m}[a_{z_1}, \, a_{z_2}]$ or$u_1 = \frac{r_1}{m} = \frac{1}{m}[-a_{z_1} \dot{z} - a_{z_2}(z - z_d)]$, which provides $z \to z_d$ at$t \to \infty$.

Similarly, we can state the synthesis problem for heading angle control. In this case, let us assume that

$$p_2 = [(\psi - \psi_d), \ \frac{d(\psi - \psi_d)}{dt}]'.$$

Let denote

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ u_2 = \tilde{\tau}_\psi.$$

Then from (4), we obtain

$$\begin{aligned} \dot{p}_2 &= A_2 p_2 + B_2 u_2, \\ p_2(0) &= p_2^0. \end{aligned} \tag{18}$$

Further, as in the previous case, we will use the following performance index

$$J_2 = \int\limits_0^\infty \left(p_2' Q_2 p_2 + u_2' R_2 u_2\right) dt,$$

where the matrices $Q_2 R_2$ and are providing solutions to the synthesis problem for rotational motion control of the quadrotor relatively axis Z by the angle ψ. Solving this synthesis problem and applying the same standard LQR synthesis procedure, we obtain the state feedback coefficients $u_2 = -K_\psi p_2$ in the form $K_\psi = [a_{\psi_1}, a_{\psi_2}]$ or $u_2 = \tilde{\tau}_\psi = -a_{\psi_1}\dot{\psi} - a_{\psi_2}(\psi - \psi_d)$, which in turn provides $\psi \to \psi_d$ at $t \to \infty$. It is worth noting here that the use of LQR synthesis for controlling a quadrotor has demonstrated its effectiveness not only for controlling the flight of a single quadrotor but also for controlling the flight of a group of quadrotors [9, 11].
As shown in [2], as time tends to infinity, from (14) and (15) we obtain $r_1 \to 0$. Then, for large values of time$T$, the value of r_1 will be sufficiently small, and then from (12) and (13) we obtain

$$\ddot{x} = -g\frac{\tan\theta}{\cos\varphi}, \tag{19}$$

$$\ddot{y} = g\tan\varphi. \tag{20}$$

Further, likewise to [3, 6], considering the angles θ, φ as small in (19) and (20), and taking into account equations (5), (6), we obtain the following relations that determine the changes of these coordinates:

$$\ddot{y} = g\varphi, \tag{21}$$

$$\ddot{\varphi} = \tilde{\tau}_\varphi, \tag{22}$$

$$\ddot{x} = -g\theta, \tag{23}$$

$$\ddot{\theta} = \tilde{\tau}_\theta. \tag{24}$$

Since in this work we consider not only climbing the quadrotor to the given altitude but also turning to a given heading angle, we require that when the drone reaches the given altitude, the x, y and coordinates, as well as the roll angle φ and the pitch angle θ, must equal zero. This requirement is ensured by solving the following problems of controller synthesis.

First, instead of (21), (22), we consider the standard model of the system in the Cauchy form:

$$\begin{aligned} \dot{p}_3 &= A_3 p_3 + B_3 u_3, \\ p_3(0) &= p_3^0, \end{aligned} \tag{25}$$

where $p_3 = [y,\ \dot{y},\ \varphi,\ \dot{\varphi}]'$- is the state vector, $u_3 = \tilde{\tau}_\varphi$- is control, the matrices A_3, B_3 have the form

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}'.$$

The performance index of this system is a standard integral quadratic criterion:

$$J_3 = \int_0^\infty \left(p_3' Q_3 p_3 + u_3' R_3 u_3\right) dt. \tag{26}$$

Choosing the weighting matrices R_3, Q_3 in the criterion (26) in the appropriate form, where these matrices provide the solution to the synthesis problem of the motion control along the y axis and rotation by the angle φ and using the standard LQR - synthesis procedure, we obtain the state feedback coefficients $u_3 = -K_y p_3$ in the form $K_y = [a_{y_1}, a_{y_2}, a_{\varphi_1}, a_{\varphi_2}]$ and this provides $y \to 0$, $\varphi \to 0$ for $t \to \infty$.

Next, consider the rotational and translational motions control (θ, x). In this case, we denote

$$p_4 = [x,\ \dot{x},\ \theta,\ \dot{\theta}]', A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}',\ \ u_4 = \tilde{\tau}_\theta.$$

Then from the subsystem (23)-(24), we get

$$\begin{aligned} &\dot{p}_4 = A_4 p_4 + B_4 u_4, \\ &p_4(0) = p_4^0. \end{aligned} \tag{27}$$

To synthesize the optimal linear controller of the problem (27) in this case, we will use the following performance criterion

$$J_4 = \int_0^\infty \left(p_4' Q_4 p_4 + u_4' R_4 u_4\right) dt,$$

where Q_4 and R_4 are weight matrices that provide a solution to the problem of LQR-synthesis of motion control along the axis x and rotation by the angle θ. As a result of applying the standard LQR -synthesis procedure, we obtain the state feedback coefficients $u_4 = -K_x p_4$ in the form $K_x = [a_{x_1}, a_{x_2}, a_{\theta_1}, a_{\theta_2}]$ and this ensures $x \to 0$, $\theta \to 0$at $t \to \infty$.

Then, substituting $u_1 = -K_z p_1$, $u_2 = -K_\psi p_2$, $u_3 = -K_y p_3$ and $u_4 = -K_x p_4$, respectively, into (17), (18), (25), and (27) and solving these Cauchy problems, we obtain solutions that are the corresponding trajectory coordinates and Euler angles of the quadrotor.

Thus, as a result of the quadrotor motion control synthesis, we have a 3-D space trajectory $(x(t), y(t), z(t))$, for which$\theta \to 0$, $x \to 0$, $y \to 0$, $\varphi \to 0$,$\psi \to \psi_d$, $z \to z_d$ at $t \to \infty$.

Now, let us look at a concrete example.

Example 2.1. Let us give the initial conditions as follows:

$$z(0) = z_0 = 0,\ z'(0) = z_0' = 0,\ \psi(0) = \psi_0 = 0,\ \psi'(0) = \psi_0' = 0,$$

$$y(0) = x_0 = 50,\ y'(0) = y_0' = 0,\ \varphi(0) = \varphi_0 = 0,\ \varphi'(0) = \varphi_0' = 0,$$

$$x(0) = x_0 = -20,\ x'(0) = x_0' = 0,\ \theta(0) = \theta_0 = 0,\ \theta'(0) = \theta_0' = 0.$$

Next, let us assume that $z_d = 200,\ \psi_d = \pi/4$. As matrices $R_i,\ Q_i,\ \ i = 1, 2, 3, 4$ we take

$$R_1 = R_2 = 1,\ \ Q_1 = Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$R_3 = 10^4,\ Q_3 = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 \cdot 10^4 & -24 \\ 6 & -12 & -24 & 36 \cdot 10^4 \end{bmatrix},\ R_4 = 1,\ Q_4 = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 & -24 \\ 6 & -12 & -24 & 36 \end{bmatrix}.$$

Note that for these data, because of applying the standard LQR synthesis procedure, we obtain feedback coefficients in the form $K_z = K_\psi = [1.0000,\quad 1.7321]$,
$K_y = [0.0100,\quad 0.1083,\quad 5.5123,\quad 6.8574]$ and $K_x = [1.0000,\quad 3.2854,\quad 29.2887,\quad 9.7251]$. Because of the calculations carried out according to the compiled program on MATLAB, we obtained that at $t \to \infty$ we have $x \to 0,\ \ \theta \to 0,\ y \to 0,\ \varphi \to 0,\ z \to z_d,\ \psi \to \psi_d$ which are confirmed by the following graphs (Fig.3, Fig.4, Fig.5. and Fig.6).

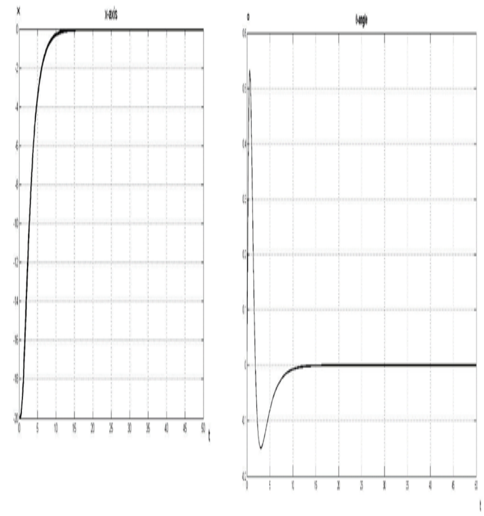


Figure 3. Coordinate x (m) and pitch angle θ (rad) vs. time t (sec).

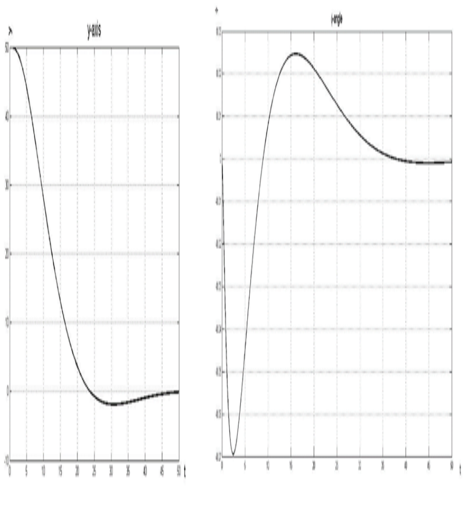


Figure 4. Coordinate y (m) and pitch angle φ (rad) vs. time t (sec).

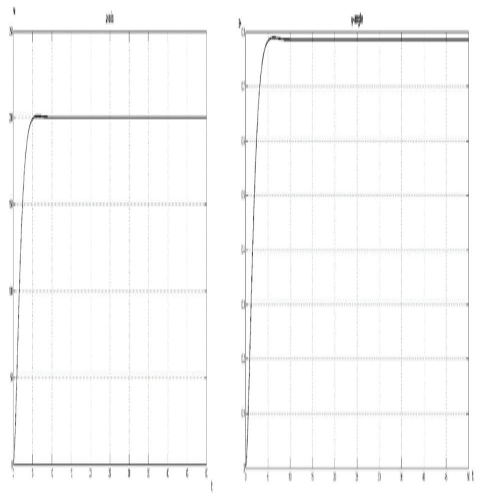


Figure 5. Coordinate z (m) and heading angle ψ (rad) vs. time t (sec).

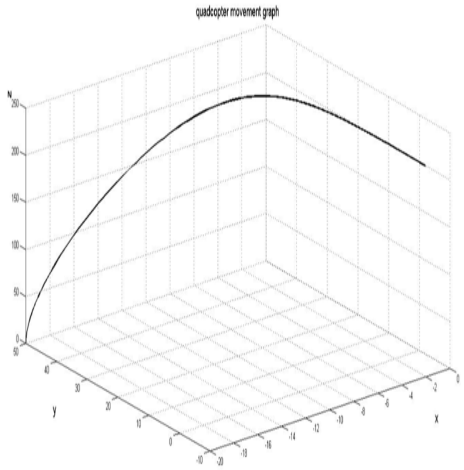


Figure 6. The motion of the center of mass during climbing to the given altitude with the heading maneuver.

3. Conclusion

(1) The paper considers and solves the problem of synthesis of the quadrotor control system in the climbing mode with a subsequent heading turn in a horizontal plane, providing the position and the attitude boundary conditions in the final point.

(2) Determination of the control laws governing the translational motion of a quadrotor along the three axes of the inertial coordinate system and the rotational motion about the vertical axis is done using LQR synthesis procedures.

(3) The obtained control laws represent simple static state feedback. The simplicity of these laws is a significant advantage in terms of their practical application.

Based on the results obtained, the flight of a quadrotor in the above-mentioned flying mode was simulated. The simulation results demonstrate the effectiveness of the proposed control laws in terms of the quality of transient processes in the above subsystems, as well as the acceptable values of the pitch and roll angles, which is very important from the point of view of the dynamic feasibility of these laws.

References

[1] Aliev, F.A., Larin, V.B., (2011), Stabilization problems for a system with output feedback, Int. Appl. Mech., 47(3), pp.3-49.

[2] Beard, R.W., McLain, T.W., (2012), Small Unmanned Aircraft: Theory and Practice, Princeton University Press, 317p.

[3] Castillo, P., Lozano, R., Dzul, A., (2005), Stabilization of a mini rotorcraft with four rotors, IEEE Control Systems Magazine, pp.45-55.

[4] Guryanov, A.E., (2014), Quadrotor control simulation, Engineering Bulletin, 8, pp.522-534.

[5] Krasovsky, A.N., Suslova, O.A., (2016), On the mathematical model of the controlled motion of a drone-quadrotor, Agrarian Bulletin of the Urals, 4, pp.55-59.

[6] Larin, V.B., Tunik, A.A., (2018), Synthesis of the quad-rotor control algorithms in the basic flight modes, TWMS J. Pure Appl. Math., 9(2), pp.147-158.

[7] Luukkonen, T., (2011), Modelling and Control of Quadrotor, Aalto University, 26p.

[8] Popov, N.I., Emelyanova, O.V., Yatsun, S.F., (2014), Simulation of quadrotor flight dynamics, Bulletin of the Voronezh Institute of the State Fire Service of the Ministry of Emergency Situations of Russia, 4(13), pp.69-75.

[9] Rinaldi, F., Chiesa, S., Quagliotti, F., (2013), Linear quadratic control for quadrotors UAVs dynamics and formation flight, J. Intell. Robot. Syst., 70, pp.203-220.

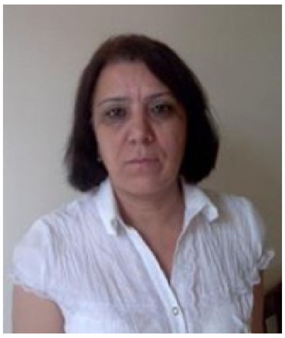
[10] Shilov, K.E., (2014), Development of an automatic control system for an unmanned aerial vehicle of a multirotor type, Proceedings of the Moscow Institute of Physics and Technology, 4, pp.139-152.

[11] Voicu, S.C., Stoica, A.M., (2021), Centralized and distributed linear quadratic design for flight formations, Incas Bulletin, 13(2), pp.175-184.

Fikret A. Aliev, for a photograph and biography, see TWMS J. Pure Appl. Math., V.1, N.1, 2010, p.12

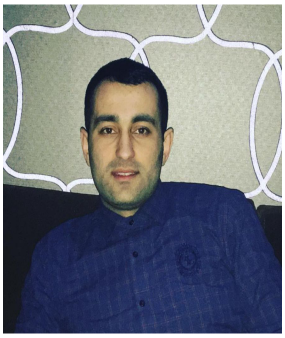
Mutallim M. Mutallimov was born in 1955 in Azerbaijan. He got his Ph.D. degree in 1984 and Doctor of Sciences degree in 2014. In present he is a Heard of Department of Computer Technology and Mathematical Modeling at the Institute of Applied Mathematics of Baku State University. His research interests are computational mathematics, theory of optimal control, computer science.

Anatoly A. Tunik, for a photograph and biography, see TWMS J. Pure Appl. Math., V.7 N.2, 2016, p.166.



Naila I. Velieva was born in 1953 in Azerbaijan. He got her Ph.D. degree in 2000 and Doctor of Sciences degree in 2021. In present she is a Leading Researcher in Department of Computer Technology and Mathematical Modeling at the Institute of Applied Mathematics of Baku State University. Her research interests are computational mathematics, theory of optimal control, computer science.

Ulviyya Z. Rasulova graduated from the Faculty of Mechanics and Information Technologies, Azerbaijan University of Architecture and Construction in 2012. She is a Ph.D. student of the Azerbaijan Architecture and Construction University in specialty "System analysis, management and information processing". After graduating university, she began her career as a Specialist of Scientific Research Department.



Samid Mirsaabov was born in 1987 in Baku. He graduated from the Mechanics-Mathematics Faculty of Baku State University in 2009. He currently works at the Institute of Control Systems of ANAS. Scientific interests include artificial intelligence, programming, mathematical modelling.