Proceedings of IAM, V.1, N.2, 2012, pp.153-162

THE INFLUENCE OF THE CARLEMAN CONDITION ON THE FREDHOLM PROPERTY OF THE BOUNDARY VALUE PROBLEM FOR CAUCHY-RIEMANN EQUATION*

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Abstract. The paper is dedicated to the investigation of Carleman condition influence on the Fredholm property of the boundary value problem for a first order elliptic equation. The obtained necessary conditions have led to the following results. If the boundary conditions of the given boundary-value problem satisfy Carleman condition then this problem can be reduced to a Fredholm integral equation of second kind whose kernel can contain only weak singularity. If Carleman condition is not satisfied then the posed problem is reduced to a Fredholm integral equation of first kind with non-singular kernel.

Keywords: boundary-value problem, non-local condition, fundamental solution, necessary conditions, regularization, Fredholm property, Carleman condition.

AMS Subject Classification: 35L02, 35L04.

1. Introduction

As is well known in the course of mathematical physics [1] and partial differential equations [2], boundary value problems are considered mainly for the Laplace equation and various elliptic equations of even order [3].

Note that in contrast to ordinary differential equations, where the number of boundary conditions coincides with the highest order of the derivatives in the equation [4], for partial differential equations the number of boundary conditions coincides with the half-order of the differential equation under consideration [1]-[3].

For the Laplace equation there is given one condition: either the Dirichlet's or Neumann's, or Poincare's. For the biharmonic equation (of fourth order) there are given two boundary conditions.

If we seek an analytic function in a bounded domain, then in the boundary condition there is given relationship between the real and imaginary parts of this function [5]-[6].

In studying the process in a nuclear reactor the mathematical model of this process is given by a linear integro-differential equation of first order in three dimensions [7]. Naturally, for such an equation, the boundary conditions of Dirichlet, Neumann or Poincare (local conditions that determine the boundary

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* Reported at the seminar of the Institute of Applied Mathematics in 13.11.2012

values of the solution) are unacceptable because such boundary conditions are given for second-order equation. Therefore, V.S.Vladimirov [7, p.51] sets the boundary conditions for almost half of the border, as the other part of the border is free of boundary conditions.

According to A.V.Bitsadze, such problems are badly set. It was said at a Bitsadze's seminar at Steklov's Institute. According to Bitsadze, the entire boundary must be the carrier for a given boundary condition.

Therefore, we have given nonlocal boundary conditions [8]-[9]. In this case, firstly, in each boundary condition the entire boundary is the carrier of this condition, and, secondly, we remove the misunderstanding between linear ordinary differential equations and partial differential equations associated with the number of boundary conditions.

Some authors in connection with the Cauchy-Riemann equations give the Dirichlet condition, but suggest that the given boundary function satisfies certain conditions [10]. We have identified these relationships as the necessary conditions containing singular integrals. Instead of regularization of these singularities (which is not so simple, because we have not only a singularity, but are on the spectrum [11]-[12]), Begehr assumes that the given boundary function satisfies these relations. Such a restriction is unacceptable.

As we noted above, we have given nonlocal boundary conditions. Under these conditions, at least two points move on the border at the same time. Then there is another difficulty. Carleman [4], [13] set that if a few points move along the boundary at the same time then the problem is badly posed if the neighboring points follow each other.

In all our papers (see website http://nihan.aliev.info which contains nearly 150 works) the Carleman condition holds true, i.e. the adjacent points move either in the opposite directions or towards each other along the boundary.

The paper is devoted to a boundary-value problem for elliptic equations of first order (Cauchy-Riemann equations) with nonlocal boundary conditions.

In the first problem when the Carleman condition holds true the problem reduces to a Fredholm integral equation of the second kind with non-singular kernel. The second problem is also posed for the Cauchy-Riemann equation with nonlocal boundary conditions, where the Carleman condition doesn't hold true. This problem is reduced to a Fredholm integral equation of the first kind with non-singular kernel.

2. Main results

If in a boundary-value problem there are at least two points on the boundary then they should not follow each other otherwise this problem is not called a "good problem" [5]. In this paper we will show how the above idea which belongs to Carleman impacts on the Fredholm property of a boundary-value problem.

Let us consider the following problems:

$$\frac{\partial u(x)}{\partial x_2}+i\frac{\partial u(x)}{\partial x_1}=0,\quad x=(x_1,x_2)\in D\subset R^2, \tag{1}$$

$$\alpha_1(x_1)u(x_1,\gamma_1(x_1))+\alpha_2(x_1)u(x_1,\gamma_2(x_1))=\alpha(x_1),\quad x_1\in[a_1,b_1] \tag{2}$$

and

$$\frac{\partial u(x)}{\partial x_2}+i\frac{\partial u(x)}{\partial x_1}=f(x),\quad x=(x_1,x_2)\in D\subset R^2, \tag{3}$$

$$\alpha_1(x_1)u(x_1,\gamma_1(x_1))+\alpha_2(x_1)u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))=0,\quad x_1\in[a_1,b_1]. \tag{4}$$

In the expressions above D is a bounded convex domain with respect to x_2-axis with boundry Γ which is a Lyapunov line. Here $i=\sqrt{-1}$ is the imaginary unit, $\alpha_1(x_1),\alpha_2(x_1)$, $\alpha(x_1)$ and $f(x)$ are given continuous functions. We assume that Γ contains two parts Γ_1 and Γ_2 whose equations can be written as follows

$$\Gamma_1: x_2=\gamma_1(x_1), \Gamma_2: x_2=\gamma_2(x_1),\quad x_1\in[a_1,b_1],$$

where Γ_1 and Γ_2 appear as a result of getting projection of Γ on the x_1-axis by using lines parallel to x_2-axis. Γ_2 is the upper part of Γ, i.e.

$$\gamma_1(x_1)\prec\gamma_2(x_1),\quad x_1\in(a_1,b_1),$$

So, $\Gamma=\Gamma_1\cup\Gamma_2,\quad [a_1,b_1]=pr_{x_1}\Gamma_1=pr_{x_1}\Gamma_2.$

Necessary conditions. It is clear that the fundamental solution of Cauchy-Riemann equation [7] is

$$U(x-\xi)=\frac{1}{2\pi}\frac{1}{x_2-\xi_2+i(x_1-\xi_1)}. \tag{5}$$

We can obtain the main relation below by using equation (2.3) and the fundamental solution (5) [14]

$$\int_\Gamma u(x)U(x-\xi)[\cos(\nu,x_2)+i\cos(\nu,x_1)]dx-\int_D f(x)U(x-\xi)dx=$$

$$=\begin{cases}u(\xi), & \xi\in D\\ \frac{1}{2}u(\xi), & \xi\in\Gamma,\end{cases} \tag{6}$$

Here ν is an external normal to the boundary Γ. The second part of the main relation is the necessary condition. Let's separate them as follows:

$$\frac{1}{2}u(\xi_1,\gamma_1(\xi_1))=\int_{\Gamma_1}u(x)U(x-\xi)|_{\xi_2=\gamma_1(\xi_1)}[\cos(\nu,x_2)+i\cos(\nu,x_1)]dx+$$

$$+\int_{\Gamma_2}u(x)U(x-\xi)|_{\xi_2=\gamma_1(\xi_1)}[\cos(\nu,x_2)+i\cos(\nu,x_1)]dx-$$

$$-\int_D f(x)U(x-\xi)|_{\xi_2=\gamma_1(\xi_1)}\,dx\,.$$

$$\frac{1}{2}u(\xi_1,\gamma_2(\xi_1))=\int_{\Gamma_1}u(x)U(x-\xi)|_{\xi_2=\gamma_2(\xi_1)}\left[\cos(\nu,x_2)+i\cos(\nu,x_1)\right]dx+$$
$$+\int_{\Gamma_2}u(x)U(x-\xi)|_{\xi_2=\gamma_2(\xi_1)}\left[\cos(\nu,x_2)+i\cos(\nu,x_1)\right]dx-$$
$$-\int_D f(x)U(x-\xi)|_{\xi_2=\gamma_2(\xi_1)}\,dx$$

In the necessary conditions above by using the expression (3) and by substituting the integrals over Γ_k, $k=1;2$, for the integrals over the interval $[a_1,b_1]$ we obtain:

$$\begin{aligned}
u(\xi_1,\gamma_1(\xi_1))=&\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1-\\
&-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{\gamma_1'(\sigma_1)-\gamma_1'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_1(x_1))}{\gamma_1'(\sigma_1)+i}dx_1+\\
&+\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_2(x_1))\left[1-i\gamma_2'(x_1)\right]}{\gamma_2(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx_1-\\
&-\frac{1}{\pi}\int_D\frac{f(x)}{x_2-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx,\quad \xi_1\in(a_1,b_1)
\end{aligned}\tag{7}$$

$$\begin{aligned}
u(\xi_1,\gamma_2(\xi_1))=&-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1+\\
&+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{\gamma_2'(\sigma_2)-\gamma_2'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_2(x_1))}{\gamma_2'(\sigma_2)+i}dx_1-\\
&-\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))\left[1-i\gamma_1'(x_1)\right]}{\gamma_1(x_1)-\gamma_2(\xi_1)+i(x_1-\xi_1)}dx_1-\\
&-\frac{1}{\pi}\int_D\frac{f(x)}{x_2-\gamma_2(\xi_1)+i(x_1-\xi_1)}dx,\quad \xi_1\in(a_1,b_1).
\end{aligned}\tag{8}$$

Here $\sigma_k=\sigma_k(x_1,\xi_1)$, $k=1;2$ is a point between x_1 and ξ_1. Thus we obtain the following

Theorem 1. Suppose D is a bounded domain convex with respect to x_2 -axis, its boundary $\Gamma = \overline{D} \setminus D$ is a Lyapunov line and $f(x)$ is a continuous function. Then each solution of equation (3) defined in the domain D satisfies the necessary conditions (7) and (8).

Remark. If we assume $f(x) \equiv 0$ in Theorem 1, then we obtain necessary conditions for the homogeneous Cauchy-Riemann equation (1).

Now, let us consider the problem (1)-(2). Each of the obtained necessary conditions includes one singular term. In order to regularize them we prove the theorem below.

Theorem 2. Under the conditions of Theorem 1, if $f(x) \equiv 0$, $\alpha_k(x_1)$, $k = 1;2$, satisfy the Hölder condition, $\alpha(x_1) \in C^{(1)}(a_1, b_1) \cap C[a_1, b_1]$ and $\alpha(a_1) = \alpha(b_1) = 0$ then we can obtain one regular condition by means of the necessary conditions (7) and (8).

Proof. Let us form the linear combination of the necessary conditions (7) and (8) so that after grouping to get singularity only under the integral whose singularity coefficient coincides with the left hand side of the boundary condition (2) and the rest of the integrals have no more than a weak singularity:

$$\begin{aligned}
&\alpha_1(\xi_1)u(\xi_1, \gamma_1(\xi_1)) - \alpha_2(\xi_1)u(\xi_1, \gamma_2(\xi_1)) = \\
&= \frac{i}{\pi}\int_{a_1}^{b_1} \frac{\alpha_1(x_1)u(x_1, \gamma_1(x_1)) + \alpha_2(x_1)u(x_1, \gamma_2(x_1))}{x_1 - \xi_1} dx_1 + \\
&+ \frac{i}{\pi}\int_{a_1}^{b_1} \frac{(\alpha_1(\xi_1) - \alpha_1(x_1))u(x_1, \gamma_1(x_1))}{x_1 - \xi_1} dx_1 + \\
&+ \frac{i}{\pi}\int_{a_1}^{b_1} \frac{(\alpha_2(\xi_1) - \alpha_2(x_1))u(x_1, \gamma_2(x_1))}{x_1 - \xi_1} dx_1 - \\
&- \frac{i\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1} \frac{\gamma_1'(\sigma_1) - \gamma_1'(x_1)}{x_1 - \xi_1} \frac{u(x_1, \gamma_1(x_1))}{\gamma_1'(\sigma_1) + i} dx_1 + \\
&+ \frac{\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1} \frac{u(x_1, \gamma_2(x_1))\left[1 - i\gamma_2'(x_1)\right]}{\gamma_2(x_1) - \gamma_1(\xi_1) + i(x_1 - \xi_1)} dx_1 - \\
&- \frac{i\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1} \frac{\gamma_2'(\sigma_2) - \gamma_2'(x_1)}{x_1 - \xi_1} \frac{u(x_1, \gamma_2(x_1))}{\gamma_2'(\sigma_2) + i} dx_1 + \\
&+ \frac{\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1} \frac{u(x_1, \gamma_1(x_1))\left[1 - i\gamma_1'(x_1)\right]}{\gamma_1(x_1) - \gamma_2(\xi_1) + i(x_1 - \xi_1)} dx_1 .
\end{aligned} \tag{9}$$

Thus, instead of two singular necessary conditions we have obtained one regular relationship (9).

Now, let us regularize the necessary conditions of the problem (3)-(4).

Theorem 3. Under the conditions of Theorem 1 if $\alpha_k(x_1),\ k=1;2,$ satisfy the Hölder condition then we can obtain one regular condition by means of the necessary conditions (7) and (8).

Proof. On the basis of (4) let us transform the necessary conditions (7) and (8) as follows:

$$
\begin{aligned}
u(\xi_1,\gamma_1(\xi_1)) &= \frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1 - \\
&-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{[\gamma_1'(\sigma_1)-\gamma_1'(x_1)]}{x_1-\xi_1}\frac{u(x_1,\gamma_1(x_1))}{\gamma_1'(\sigma_1)+i}dx_1 + \\
&+\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))\left[1-i\gamma_2'(a_1+b_1-x_1)\right]}{\gamma_2(a_1+b_1-x_1)-\gamma_1(\xi_1)+i(a_1+b_1-x_1-\xi_1)}dx_1 - \\
&-\frac{1}{\pi}\int_D\frac{f(x)}{x_2-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
u(a_1+b_1-\xi_1,\gamma_2(a_1+b_1-\xi_1)) &= \frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))}{x_1-\xi_1}dx_1 - \\
&-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))}{\gamma_2'(\sigma_2(a_1+b_1-x_1,a_1+b_1-\xi_1))+i}\times \\
&\times\frac{\gamma_2'(\sigma_2(a_1+b_1-x_1,a_1+b_1-\xi_1))-\gamma_2'(a_1+b_1-x_1)}{x_1-\xi_1}dx_1 - \\
&-\frac{1}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))\left[1-i\gamma_1'(x_1)\right]}{\gamma_1(x_1)-\gamma_2(a_1+b_1-\xi_1)+i(x_1-a_1-b_1+\xi_1)}dx_1 - \\
&-\frac{1}{\pi}\int_D\frac{f(x)}{x_2-\gamma_2(a_1+b_1-\xi_1)+i(x_1-a_1-b_1+\xi_1)}dx.
\end{aligned}
\tag{11}
$$

Here we substitute $x_1=a_1+b_1-\eta_1$ into the terms including $u(x_1,\gamma_2(x_1))$ and then write x_1 instead of η_1.

Let us form such a linear combination of (10) aned (11) and group it so that the coefficient of the singular term coincides with the left hand side of the boundary condition (4) and, therefore, the singularity vanishes:

$$
\begin{aligned}
&\alpha_1(\xi_1)u(\xi_1,\gamma_1(\xi_1))+\alpha_2(\xi_1)u(a_1+b_1-\xi_1,\gamma_2(a_1+b_1-\xi_1))=\\
&=\frac{i}{\pi}\int_{a_1}^{b_1}\frac{\alpha_1(x_1)u(x_1,\gamma_1(x_1))+\alpha_2(x_1)u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))}{x_1-\xi_1}dx_1+\\
&+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_1(\xi_1)-\alpha_1(x_1))u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1+\\
&+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_2(\xi_1)-\alpha_2(x_1))u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))}{x_1-\xi_1}dx_1-\\
&-\frac{i\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{\gamma_1'(\sigma_1)-\gamma_1'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_1(x_1))}{\gamma_1'(\sigma_1)+i}dx_1+\\
&+\frac{\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))\left[1-i\gamma_2'(a_1+b_1-x_1)\right]}{\gamma_2(a_1+b_1-x_1)-\gamma_1(\xi_1)+i(a_1+b_1-x_1-\xi_1)}dx_1-\\
&-\frac{\alpha_1(\xi_1)}{\pi}\int_D\frac{f(x)}{x_2-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx-
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&-\frac{i\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(a_1+b_1-x_1,\gamma_2(a_1+b_1-x_1))}{\gamma_2'(\sigma_2(a_1+b_1-x_1,a_1+b_1-\xi_1))+i}\times\\
&\times\frac{\gamma_2'(\sigma_2(a_1+b_1-x_1,a_1+b_1-\xi_1))-\gamma_2'(a_1+b_1-x_1)}{x_1-\xi_1}dx_1-\\
&-\frac{\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))\left[1-i\gamma_1'(x_1)\right]}{\gamma_1(x_1)-\gamma_2(a_1+b_1-\xi_1)+i(x_1-a_1-b_1+\xi_1)}dx_1-\\
&-\frac{\alpha_2(\xi_1)}{\pi}\int_D\frac{f(x)}{x_2-\gamma_2(a_1+b_1-\xi_1)+i(x_1-a_1-b_1+\xi_1)}dx.
\end{aligned}
$$

If we substitute the necessary condition (4) into the first integral on the right hand side of (12) then the expression (12) becomes regular.

Now let us prove the next theorem:

Theorem 4. Under the conditions of Theorem 2 if $\alpha_k(x_1)\neq 0,\ k=1;2\ x_1\in[a_1,b_1]$ then the problem (1)-(2) is a Fredholm problem.

Proof. Indeed, by taking the regularized expression (9) and the boundary condition (2) together we can obtain a system of integral Fredholm equations of the second kind with respect to $u(x_1,\gamma_k(x_1)),\ k=1,2$ with weak singularity in the kernels:

$$u(\xi_1,\gamma_1(\xi_1))=\frac{1}{2\alpha_1(\xi_1)}\left\{\alpha(\xi_1)+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{\alpha_1(x_1)u(x_1,\gamma_1(x_1))+\alpha_2(x_1)u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1+\right.$$
$$+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_1(\xi_1)-\alpha_1(x_1))u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1+\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_2(\xi_1)-\alpha_2(x_1))u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1-$$
$$-\frac{i\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{\gamma_1'(\sigma_1)-\gamma_1'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_1(x_1))}{\gamma_1'(\sigma_1)+i}dx_1+\frac{\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_2(x_1))\left[1-i\gamma_2'(x_1)\right]}{\gamma_2(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx_1-$$
$$\left.-\frac{i\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{\gamma_2'(\sigma_2)-\gamma_2'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_2(x_1))}{\gamma_2'(\sigma_2)+i}dx_1+\frac{\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))\left[1-i\gamma_1'(x_1)\right]}{\gamma_1(x_1)-\gamma_2(\xi_1)+i(x_1-\xi_1)}dx_1\right\}.$$

$$u(\xi_1,\gamma_2(\xi_1))=\frac{1}{2\alpha_2(\xi_1)}\left\{\alpha(\xi_1)-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{\alpha_1(x_1)u(x_1,\gamma_1(x_1))+\alpha_2(x_1)u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1-\right.$$
$$-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_1(\xi_1)-\alpha_1(x_1))u(x_1,\gamma_1(x_1))}{x_1-\xi_1}dx_1-\frac{i}{\pi}\int_{a_1}^{b_1}\frac{(\alpha_2(\xi_1)-\alpha_2(x_1))u(x_1,\gamma_2(x_1))}{x_1-\xi_1}dx_1+$$
$$+\frac{i\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{\gamma_1'(\sigma_1)-\gamma_1'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_1(x_1))}{\gamma_1'(\sigma_1)+i}dx_1-\frac{\alpha_1(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_2(x_1))\left[1-i\gamma_2'(x_1)\right]}{\gamma_2(x_1)-\gamma_1(\xi_1)+i(x_1-\xi_1)}dx_1+$$
$$\left.+\frac{i\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{\gamma_2'(\sigma_2)-\gamma_2'(x_1)}{x_1-\xi_1}\frac{u(x_1,\gamma_2(x_1))}{\gamma_2'(\sigma_2)+i}dx_1-\frac{\alpha_2(\xi_1)}{\pi}\int_{a_1}^{b_1}\frac{u(x_1,\gamma_1(x_1))\left[1-i\gamma_1'(x_1)\right]}{\gamma_1(x_1)-\gamma_2(\xi_1)+i(x_1-\xi_1)}dx_1\right\}.$$

In the case of the second problem we see that the left hand side of the regular expression (12) is equal to zero since it is the same as the boundary condition (4). So there remains no unknown out of the integral, i.e. this expression is the first Fredholm integral. That is why this problem can't be reduced to the system of second Fredholm integral equations.

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Koşi-Riman tənliyi üçün sərhəd məsələsinin fredholmluğuna Karleman şərtinin təsiri

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XÜLASƏ

Məqalə birinci tərtib elliptik tip tənlik üçün sərhəd məsələsinin fredholmluğuna Karleman şərtinin təsirinə həsr olunub. Alınan zəruri şərtlər Karleman şərti ödənildikdə ikinci növ, ödənilmədikdə isə birinci növ Fredholm tipli integral tənliyə gətirilir.

Açar sözlər: sərhəd məsələsi, qeyri-lokal şərtlər, fundamental həll, zəruri şərtlər, requlyarlaşdırma, fredholmluluq, Karleman şərti.

Влияние условия Карлемана на фредгольмовость граничной задачи для уравнения Коши-Римана

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РЕЗЮМЕ

Статья посвящена влиянию условия Карлемана на фредгольмовость граничной задачи для уравнения Коши-Римана. В работе показано, что при выполнении полученных необходимых условий существования решения и при выполнении условия Карлемана задача сводится интегральному уравнению Фредгольма второго рода, а при невыполнении–к интегральному уравнению Фредгольма первого рода.

Ключевые слова: краевая задача, нелокальные условия, фундаментальное решение, необходимые условия, регуляризация, фредгольмовость, условия Карлемана.