Bahram M. Askerov Sophia R. Figarova

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Thermodynamics, Gibbs Method and Statistical Physics of Electron Gases



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Bahram M. Askerov Sophia R. Figarova

Thermodynamics, Gibbs Method and Statistical Physics of Electron Gases

With 101 Figures



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Springer Series on Atomic, Optical, and Plasma Physics ISSN 1615-5653 ISBN 978-3-642-03170-0 e-ISBN 978-3-642-03171-7 DOI 10.1007/978-3-642-03171-7 Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2009936788

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Cover design: SPi Publisher Services

Printed on acid-free paper

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Preface

Thermodynamics and statistical physics study the physical properties (mechanical, thermal, magnetic, optical, electrical, etc.) of the macroscopic system. The tasks and objects of study in thermodynamics and statistical physics are identical. However, the methods of investigation into macroscopic systems are different.

Thermodynamics is a *phenomenological theory*. It studies the properties of bodies, without going into the mechanism of phenomena, i.e., not taking into consideration the relation between the internal structure of substance and phenomena, it generalizes experimental results. As a result of such a generalization, postulates and laws of thermodynamics made their appearance. These laws make it possible to find general relations between the different properties of macroscopic systems and the physical events occurring in them.

Statistical physics is a microscopic theory. On the basis of the knowledge of the type of particles a system consists of, the nature of their interaction, and the laws of motion of these particles issuing from the construction of substance, it explains the properties being observed on experiment, and predicts the new properties of systems. Using the laws of classical or quantum mechanics, and also the theory of probability, it establishes qualitatively new statistical appropriatenesses of the physical properties of macroscopic systems, substantiates the laws of thermodynamics, determines the limits of their applicability, gives the statistical interpretation of thermodynamic parameters, and also works out methods of calculations of their means. The Gibbs method is based on statistical physics. This method is the most canonical. Therefore, in this book, the exposition of the Gibbs method takes an important place.

Results, stemming from phenomenological thermodynamics, bear the general character and can be applied to any macroscopic systems; however, the internal mechanism of physical phenomena and properties, being observed in the experiments, is not disclosed. In other words, thermodynamics only describes the phenomena and establishes the relation between them, but does not answer the question why it happens just so. Statistical physics relates the properties of bodies to their internal construction, creates the microscopic theory of physical phenomena, and answers the question why it happens just so. The disadvantage of this method resides in the fact that results, obtained here, bear a particular character and are right only in frames of the considered model of the structure of substance.

Thermodynamics and statistical physics study not only equilibrium systems, but also systems in which specified currents and flows (the electric current, flow of energy and substance) exist. In this case, the theory is called *thermodynamics of non-equilibrium systems* or *kinetics*. Kinetics originates from the Boltzmann equation (1872) and has continued developing up to the present time.

The development of phenomenological thermodynamics started in the first half of the nineteenth century.

The first law of thermodynamics was discovered by the German physiologist Julius Robert von Mayer (1842) and the English physicist James Prescott Joule (1843). They showed the equivalence of heat and mechanical work. The first law of thermodynamics is a law of conservation of energy for closed processes. In 1847, the German physicist and physiologist Hermann von Helmholtz generalized this law for any non-closed thermodynamic processes.

The second law of thermodynamics was discovered independently by both the German physicist Rudolf Clausius (1850) and the English physicist William Thomson (Lord Kelvin). They introduced in the theory a new function of state – entropy, in the statistical sense, and discovered the law of increasing entropy.

The third law of thermodynamics was discovered in 1906 by the German physicist-chemist Walther Nernst. According to this law, entropy of all systems independently of external parameters tends to the identical value (zero) as temperature approaches the absolute zero.

Note that the first law of thermodynamics is a law about energy, and the second and the third ones are about entropy.

The founders of thermodynamics are J.R. von Mayer, J.P. Joule, H. von Helmholtz, R. Clausius, W. Kelvin, and W. Nernst.

Statistical physics received its development only in the last quarter of the nineteenth century. The founders of classical statistical physics are R. Clausius, J.C. Maxwell, L. Boltzmann, and J.W. Gibbs. The height of development of classical statistical physics is the method of Josiah Willard Gibbs (1902).

The application of classical statistics to many problems provided results, though not coinciding with the experimental facts of that time. Black radiation (thermodynamics of a photon gas), heat capacity of metals, Pauli paramagnetism, etc. can serve as examples. These difficulties of classical statistics were circumvented only after the rise of quantum mechanics (L. de Broglie, W. Heisenberg, E. Schrödinger, and P. Dirac) and quantum statistics, created on its basis (E. Fermi, P. Dirac, S.N. Bose, A. Einstein) during 1924–1926.

The method of thermodynamic functions and potentials, and also the Gibbs statistical method or the methods of free energy, being the key-note of the book, occupy an important place. It is shown that of all the thermodynamic functions, the most important are the function of free energy and grand thermodynamic potential, which are determined from the Gibbs canonical distribution. It is expalined that the basic postulate of statistical physics – the *microcanonical distribution* of isolated systems – is based on the statistical theory of the macroscopic properties of a system, from which all canonical distribution stems.

Understanding free energy and grand thermodynamic potential, it is easy to determine entropy, thermal and caloric equations of state, and also all thermodynamic coefficients, measured by testing. To do this in the case of classical systems, it is sufficient to know the Hamilton function – energy as a function of coordinates and impulses of particles of the system, forming it, and for quantum systems, it is the energy spectrum, i.å., the dependence of energy on quantum numbers. It is also an essence of the Gibbs method, which is applied to ideal and non-ideal gases, and also to a crystalline solid. The exposition of the Fermi-Dirac and Bose–Einstein quantum statistics and its application to different quantum gases occupy a large place. It is shown how the difficulties of classical statistics, associated with its application to an electron gas in metals, are circumvented. The statistics of the electron gases are considered in detail in this book.

A separate chapter is devoted to the statistical theory of thermodynamic properties of an electron gas in a quantizing magnetic field. Note that the investigation of properties of an electron gas in extremal conditions, in particular, at ultra-low temperatures and in strong quantizing magnetic fields, is one of the actual tasks of contemporary physics.

In the last chapter, on the basis of the Boltzmann kinetic equation, the electron gas in metals and semiconductors is considered in a nonequilibrium state. Nonequilibrium processes are associated with charge carrier motion in a crystal under external disturbances such as the electric field and the temperature gradient in the magnetic field. They include electric conductivity, thermoelectric, galvanomagnetic, and thermomagnetic effects.

Baku, November 2009 Bahram M. Askerov Sophia R. Figarova

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2010. XII, 374 p. 202 illus., 101 in color. (Springer Series on Atomic, Optical, and Plasma Physics, Vol. 57) Hardcover

- ▶ 119,95€
- ▶ \$159.00
- SFr. 186.50
- ▶ £108.00

ISBN 978-3-642-03170-0

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This book deals with theoretical thermodynamics and the statistical physics of electron and particle gases. While treating the laws of thermodynamics from both classical and quantum theoretical viewpoints, it posits that the basis of the statistical theory of macroscopic properties of a system is the microcanonical distribution of isolated systems, from which all canonical distributions stem. To calculate the free energy, the Gibbs method is applied to ideal and non-ideal gases, and also to a crystalline solid. Considerable attention is paid to the Fermi-Dirac and Bose-Einstein quantum statistics and its application to different quantum gases, and electron gas in both metals and semiconductors is considered in a nonequilibrium state. A separate chapter treats the statistical theory of thermodynamic properties of an electron gas in a quantizing magnetic field.... more on http://springer.com/978-3-642-03170-0

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