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# Electron Transport Phenomena in Semiconductors

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## **ELECTRON TRANSPORT PHENOMENA IN SEMICONDUCTORS**

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## Foreword

Kinetic properties are known to underlie numerous technical applications of semiconductors. Besides, these properties are sensitive to the dispersion laws of the current carriers and the nature of the interaction of the carriers with various defects of the crystal lattice. Therefore, many conventional methods of investigating semiconducting materials are based on the study of different kinetic effects. They become especially efficient under some extreme conditions: at low temperatures, in strong magnetic fields, in semiconductors with a highly nonparabolic band, etc. Satisfactory and reliable results are obtained when an integral investigation is carried out and the conclusions of the theory of electron transport phenomena are taken into account. However, in the publications that are available only one or two chapters are being devoted to the theory of kinetic effects.

The present book deals with a systematic and detailed description of the linear theory of stationary electron transport phenomena in semiconductors. Both the classical and quantum theories of galvano- and thermomagnetic phenomena are set forth. Arbitrary isotropic and anisotropic nonparabolic bands as well as bands of the type of hole germanium are described. The process of entrainment of charge carriers by phonons in nonquantizing magnetic field is taken into consideration. A scattering theory with the influence of the Bloch amplitude taken into account is presented in detail.

Three important chapters constitute the main portion of the book. They are the statistics of current carriers in semiconductors, the classical theory and the quantum theory of electron transport phenomena.

The statistics of current carriers when there is no energy spectrum quantization is discussed in the second chapter. The carrier statistics in a quantizing

magnetic field and in the case of dimensional quantization is described in chapters 6 and 7 respectively.

Chapters 3 to 5 deal with the classical theory of electron transport phenomena which is based on the solution of the Boltzmann equation. The applicability limits of the Boltzmann equation and its solution are analyzed in detail. The theory of charge carrier scattering in semiconductors with an arbitrary isotropic band is presented. The relaxation time and mobility, taking account of the transitions between bands of light and heavy holes, are calculated for semiconductors of the type of hole germanium. Chapter 5 presents the transport phenomena in multivalley semiconductors where scattering anisotropy is also accounted for. The results of this chapter can be applied to the electron germanium and silicon as well as to semiconductor compounds of lead chalcogenide. It is known that the conduction band in lead chalcogenide is not only anisotropic but also nonparabolic.

The quantum theory of electron transport phenomena is discussed in chapter 6, where the galvano- and thermomagnetic effects in a transverse quantizing magnetic field in semiconductors with an isotropic band are investigated. The conditions of the Shubnikov and de Gaaz oscillations and magnetophonon resonance are considered. Special attention is given to the thermoelectromotive force in the quantum region of magnetic fields and to the influence of band nonparabolicity.

The last chapter deals with the classical and quantum dimensional effects. The Boltzmann equation, taking account of the boundary conditions for the distribution function, is solved for films whose thicknesses are comparable with the mean free path of charge carriers. Some general compact expressions are obtained for the conductivity tensors in films. The possibility of negative magnetoresistance in films with an isotropic band is shown in the case of a completely degenerate electron gas. Dimensional quantization in films is discussed in the concluding section where the thermoelectromotive force is considered in a strong transverse magnetic field.

All transport phenomenon problems considered here are reduced to particular formulae, and for the sake of convenience of application to analysis of experimental results conditions are specified for the case when they can be used.

The references to original papers concern mainly theoretical investigations and the list for each chapter is given at the end of each chapter. These lists do not lay claim to completeness.

I consider it my pleasant duty to express sincere acknowledgment to V.L.

Bonch-Bruyevich for his valuable remarks and advice while discussing the manuscript which improved its contents in many respects.

I am also indebted to B.I. Kuliyeu and S.R. Figarova who looked through the manuscript, as well as for their participation in the discussions on some aspects of the theory of transport phenomena in films.

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**B.M. Askerov, Baku, 1991.**

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## ELECTRON TRANSPORT PHENOMENA IN SEMICONDUCTORS

by B M Askerov (*Baku State Univ.*)



This book contains the first systematic and detailed exposition of the linear theory of the stationary electron transport phenomena in semiconductors. Arbitrary isotropic and anisotropic nonparabolic bands as well as p-Ge-type bands are considered. Phonon drag effect are taken account of in an arbitrary nonquantizing magnetic field. Scattering theory is discussed in detail with account taken of the Bloch wave functions effect. Transport phenomena in the quantizing magnetic field are studied as well as the size effects in thin films. Band structures of the semiconductors and semiconductor compounds of interest are also considered.

The main part of the book deals with the three important problems: charge carrier statistics in a semiconductor, classical and quantum theory of the electron transport phenomena. All the theoretical results considered as well as the validity conditions are presented in the form which may be directly used to interpret experimental data.

### Contents:

- Energy Spectrum of Charge Carriers in Semiconductors
- Statistics of Charge Carriers in Semiconductors
- Solution of Boltzmann Equation, Scattering Mechanisms
- Electron Transport Phenomena in Semiconductors with Isotropic Band
- Transport Phenomena in Semiconductors with Anisotropic Nonparabolic Band, Anisotropic Scattering
- Transport Phenomena in Quantizing Magnetic Fields
- Electron Transport Phenomena in Semiconductive Films. Size-Dependent Effects

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